A Formulary for Mathematics

A collection of
Formulas, Facts and Figures
often needed in
mathematics

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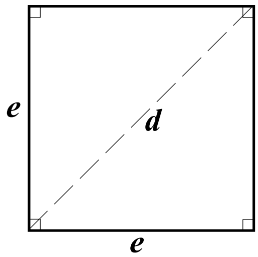
Square



e = **e**dge length
d = **d**iagonal length
P = **p**erimeter length
A = **a**rea

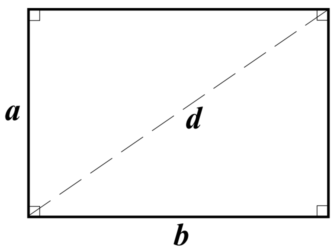
$P = 4 \times e$ $\quad P = 4 \times \sqrt{A}$ $\quad P = 2 \times d \times \sqrt{2}$

$A = e^2$ $\quad A = d^2 \div 2$ $\quad A = P^2 \div 16$

$d = e \times \sqrt{2}$ $\quad d = \sqrt{2 \times A}$ $\quad d = \dfrac{P \times \sqrt{2}}{4}$

$e = \sqrt{A}$ $\quad e = P \div 4$ $\quad e = \dfrac{d \times \sqrt{2}}{2}$

Oblong



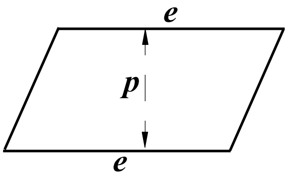
a, b = edge lengths
d = **d**iagonal length
P = **p**erimeter length
A = **a**rea

$P = 2 \times (a + b)$ $\quad a = \dfrac{P}{2} - b$ $\quad b = \dfrac{P}{2} - a$

$d = \sqrt{a^2 + b^2}$ $\quad a = \sqrt{d^2 - b^2}$ $\quad b = \sqrt{d^2 - a^2}$

$A = a \times b$ $\quad a = A \div b$ $\quad b = A \div a$

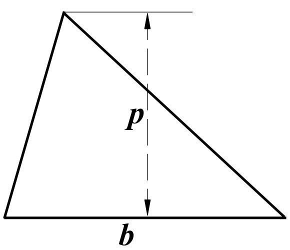
Parallelogram



e = **e**dge lengths of two parallel edges
p = **p**erpendicular distance between them
A = **a**rea

$A = p \times e$

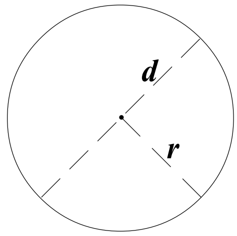
Triangle



b = **b**ase length
p = **p**erpendicular height
A = **a**rea

$A = p \times b$

Circle



r = **r**adius length
d = **d**iameter length
C = **c**ircumference length
A = **a**rea

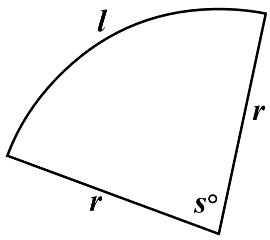
$C = 2 \times \pi \times r$ $\quad C = \pi \times d$ $\quad C = 2 \times \sqrt{A \times \pi}$

$A = 2 \times \pi \times r^2$ $\quad A = \dfrac{\pi \times d^2}{4}$ $\quad A = \dfrac{C^2}{4 \times \pi}$

$d = 2 \times r$ $\quad d = 2 \times \sqrt{\dfrac{A}{\pi}}$ $\quad d = C \div \pi$

$r = d \div 2$ $\quad r = \sqrt{\dfrac{A}{\pi}}$ $\quad r = \dfrac{C}{2 \times \pi}$

Sector



s° = sector angle *(in degrees)*
l = **l**ength of arc
r = **r**adius of circle
A = **a**rea of sector

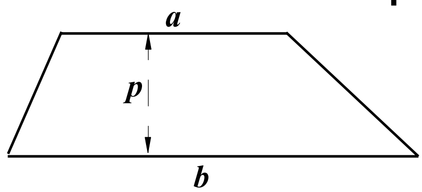
$l = \pi \times r \times s° \div 180$

$A = \pi \times r^2 \times s° \div 360$ $\quad A = r \times l \div 2$

$r = 2 \times A \div l$ $\quad l = 2 \times A \div r$

$r = \dfrac{180\, l}{\pi s°}$ $\quad s° = \dfrac{180\, l}{\pi r}$ $\quad s° = \dfrac{360\, A}{\pi r^2}$

Trapezium



a, b = edge lengths of two parallel edges
p = **p**erpendicular distance between them
A = **a**rea

$A = p \times (a + b) \div 2$

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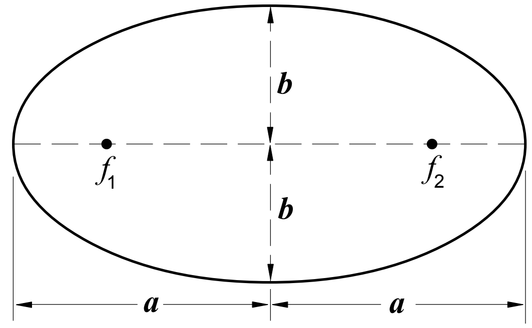
Ellipse



a = half-length of **major** axis

b = half-length of **minor** axis

C = **c**ircumference

A = **a**rea

e = **e**ccentricity

d = **d**istance between foci f_1 and f_2

$$A = \pi \times a \times b$$

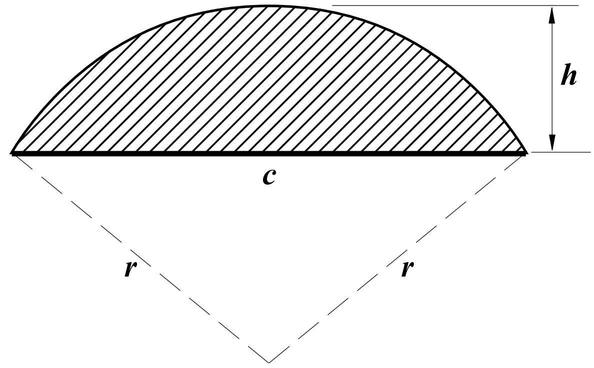
$$C = \pi \times (a + b) \quad \text{[good approximation]}$$

$$C = \pi \times \sqrt{(a^2 + b^2) \times 2} \quad \text{[better approximation]}$$

$$C = \pi \times [3(a+b) - \sqrt{(a+3b)(3a+b)}\,] \quad \text{[best approximation]}$$

$$e = \sqrt{1 - \frac{b^2}{a^2}} \qquad d = 2 \times a \times e$$

Segment



r = **r**adius of circle from which segment is cut

s° = **s**egment angle at centre of circle *(in degrees)*

c = **c**hord length of segment

h = **h**eight of segment

A = **a**rea of segment

$$A = \frac{s° \pi r^2}{360} - \frac{r^2 \sin s°}{2} \qquad r = \frac{c^2 + 4h^2}{8h}$$

$$c = 2 \times r \sin \frac{s°}{2} \qquad c = 2 \times \sqrt{h \times (2r - h)}$$

$$h = r\,(1 - \sec \frac{s°}{2})$$

Regular Polygons

If the edge length of the polygon is ***e*** then

area of the **polygon** is given by $A = e^2 \times$ A-factor

radius of the **circumcircle** is given by $R = e \times$ C-factor

radius of the **incircle** is given by $r = e \times$ I-factor

The necessary factors are to be found in the table below.

The size of its interior vertex angle (in degrees) is also given

No. of edges	Name	A-factor	C-factor	I-factor	Vertex Angle
3	triangle	0.43301	0.57735	0.28868	60
4	quadrilateral	1	0.70711	0.5	90
5	pentagon	1.7205	0.85065	0.68819	108
6	hexagon	2.5981	1	0.86603	120
7	heptagon	3.6339	1.1524	1.0383	128.57
8	octagons	4.8284	1.3066	1.2071	135
9	nonagon	6.1818	1.4619	1.3737	140
10	decagon	7.6942	1.6180	1.5388	144
11	undecagon	9.3656	1.7747	1.7028	147.27
12	dodecagon	11.196	1.9319	1.8660	150

Except for values which are exact, all others are given to 5 significant figures.

For any regular polygon having ***n*** edges the factors are given by

$$\text{A-factor} = \frac{n}{4} \cot \frac{180}{n} \qquad \text{C-factor} = \frac{1}{2} \operatorname{cosec} \frac{180}{n} \qquad \text{I-factor} = \frac{1}{2} \cot \frac{180}{n}$$

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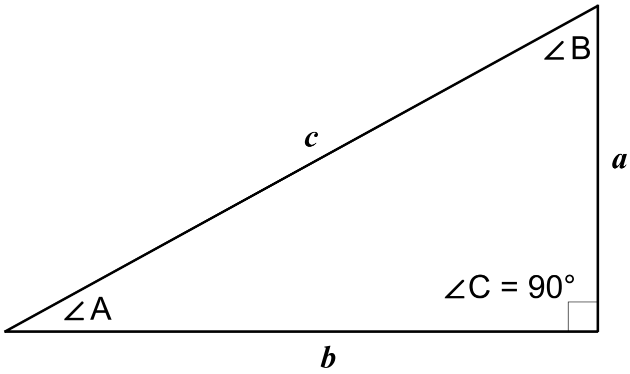

Take care to match given data to the correct letters

Given	Use the formula from the appropriate box below to find				
	a	b	c	∠A	∠B
a b			$c = \sqrt{a^2 + b^2}$	$\tan A = a \div b$	$\tan B = b \div a$
a c		$b = \sqrt{c^2 - a^2}$		$\sin A = a \div c$	$\cos B = a \div c$
b c	$a = \sqrt{c^2 - b^2}$			$\cos A = b \div c$	$\sin B = b \div c$
a ∠A		ê `	$c = a \div \sin A$		B = 90° − A
a ∠B		$b = a \times \tan B$	$c = a \div \cos B$	A = 90° − B	
b ∠A	$a = b \times \tan A$		$c = b \div \cos A$		B = 90° − A
b ∠B	$a = b \div \tan B$		$c = b \div \sin B$	A = 90° − B	
c ∠A	$a = c \times \sin A$	$b = c \times \cos A$			B = 90° − A
c ∠B	$a = c \times \cos B$	$b = c \times \sin B$		A = 90° − B	

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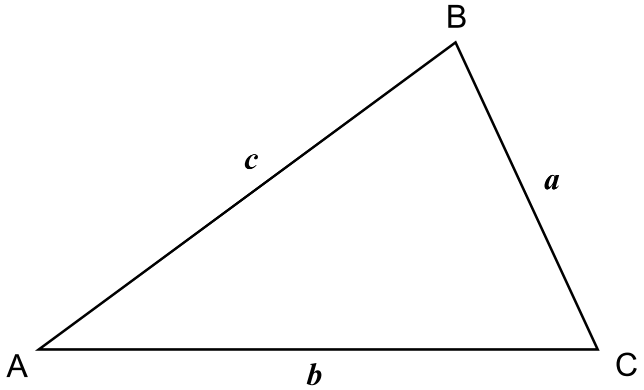


The semi-perimeter is given by $s = (a + b + c) \div 2$ which is more usually written as

$$s = \frac{a+b+c}{2}$$

Δ is the symbol for **area**

Area = $\frac{1}{2}ab\sin C$ *or* $\frac{1}{2}ac\sin B$ *or* $\frac{1}{2}bc\sin A$ *or* $\sqrt{s(s-a)(s-b)(s-c)}$

Sine Rule $\dfrac{a}{\sin A} = \dfrac{b}{\sin B} = \dfrac{c}{\sin C}$

Cosine Rule

$a^2 = b^2 + c^2 - 2bc\cos A$ *or* $\cos A = (b^2 + c^2 - a^2) \div 2bc$

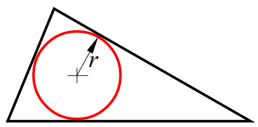
$b^2 = a^2 + c^2 - 2ac\cos B$ *or* $\cos B = (a^2 + c^2 - b^2) \div 2ac$

$c^2 = a^2 + b^2 - 2ab\cos C$ *or* $\cos C = (a^2 + b^2 - c^2) \div 2ab$

Tangent Rule $\tan\dfrac{B-C}{2} = \dfrac{b-c}{b+c}\cot\dfrac{A}{2}$

Half-angle Formulas $\sin\dfrac{A}{2} = \sqrt{\dfrac{(s-b)(s-c)}{bc}}$ $\cos\dfrac{A}{2} = \sqrt{\dfrac{s(s-a)}{bc}}$ $\tan\dfrac{A}{2} = \sqrt{\dfrac{(s-b)(s-c)}{s(s-a)}}$

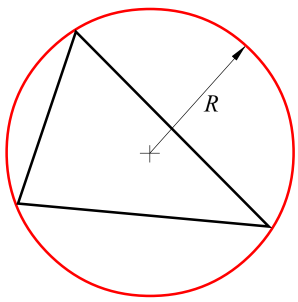
Inscribed Circle



Δ = Area

radius $r = \dfrac{\Delta}{s}$

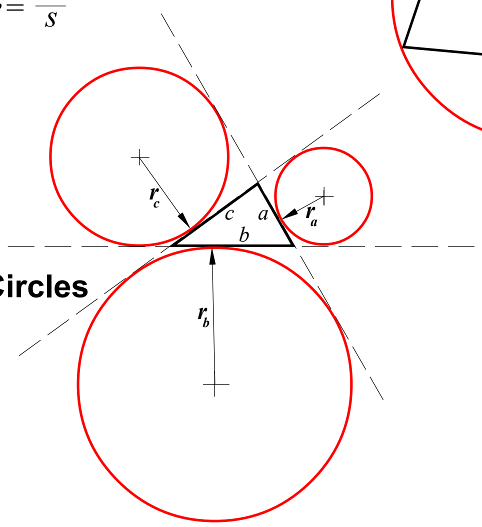
Circumscribed Circle



Radius $R = \dfrac{abc}{4\Delta}$

$R = \dfrac{a}{2\sin A}$ *or* $\dfrac{b}{2\sin B}$ *or* $\dfrac{c}{2\sin C}$

Escribed Circles



The different radii needed for the three possible escribed circles are identified by the letters of the edge on which each circle is placed $\mathbf{r}_a$ $\mathbf{r}_b$ $\mathbf{r}_c$

$\mathbf{r}_a = \dfrac{\Delta}{s-a}$ $\mathbf{r}_b = \dfrac{\Delta}{s-b}$ $\mathbf{r}_c = \dfrac{\Delta}{s-c}$

All the above formulas are **cyclic**
That is, the six variables (*a, b, c,* A, B, C) can be changed around as long as the pattern of the formula is kept.
This is best seen in the **Cosine Rule** where all three possible variations are given, and the pattern is clear.

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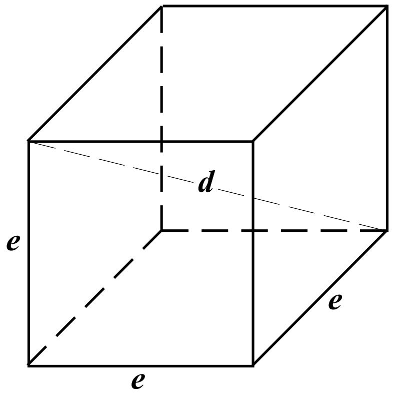
Cube



e = **e**dge length
d = **d**iagonal length
S = **s**urface area
V = **v**olume

$S = 6 \times e^2$ $\quad V = e^3$ $\quad d = e \times \sqrt{3}$

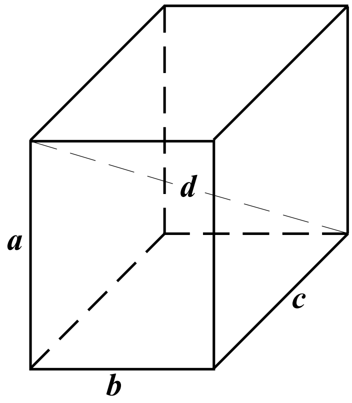
$e = \sqrt{\frac{S}{6}}$ $\quad e = \sqrt[3]{V}$ $\quad e = d \div \sqrt{3}$

$S = 6 \times \sqrt[3]{V^2}$ $\quad S = 2 \times d^2$

$V = \sqrt{\frac{S^3}{216}}$ $\quad V = \frac{d^3 \times \sqrt{3}}{9}$

$d = e\sqrt{\frac{S}{2}}$

Cuboid



a, b, c = edge lengths
d = **d**iagonal length
S = **s**urface area
V = **v**olume

$V = a \times b \times c$

$d = \sqrt{a^2 + b^2 + c^2}$

$S = 2 \times (ab + bc + bc)$

$a = \frac{V}{bc}$ $\quad b = \frac{V}{ac}$ $\quad c = \frac{V}{ab}$

Regular Polyhedrons

Associated with any regular convex polyhedron are two particular spheres.
A **circumsphere** is the sphere drawn around the *outside* of a regular convex polyhedron so as as to touch every vertex of that polyhedron.
An **insphere** is the sphere drawn around the *inside* of a regular convex polyhedron so as as to touch every face of that polyhedron.
If the edge length of the polyhedron is e then

- **area** of the surface of the **polyhedron** is given by $e^2 \times$ A-factor
- **volume** of the **polyhedron** is given by $e^3 \times$ V-factor
- **radius** of the **circumsphere** is given by $e \times$ C-factor
- **radius** of the **insphere** is given by $e \times$ I-factor

The necessary factors are to be found in the table below.
The size of the dihedral angle (in degrees) between faces is also given

No. of faces	Name	A-factor	V-factor	C-factor	I-factor	Dihedral Angle
4	tetrahedron	1.73205	0.117851	0.612372	0.204124	70.5333
6	cube	6	1	0.866025	0.5	90
8	octahedron	3.46410	0.471405	0.707107	0.408248	109.467
12	dodecahedron	20.6458	7.66312	1.40126	1.11352	116.565
20	icosahedron	8.66025	2.18170	0.951057	0.755761	138.190

Except for values which are exact, all others are given to 6 significant figures.

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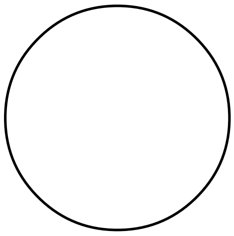
Sphere



r = **r**adius

d = **d**iameter

C = **c**ircumference

A = **a**rea of surface

V = **v**olume

$C = 2 \times \pi \times r$ *or* $\pi \times d$

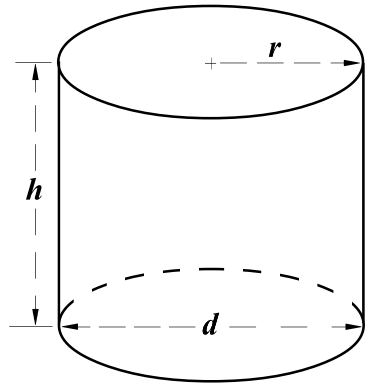
$A = 4 \times \pi \times r^2$ *or* $\pi \times d^2$

$V = 4 \times \pi \times r^3 \div 3$ *or* $\pi \times d^3 \div 6$

$d = 2 \times r$ *or* $\sqrt{\frac{A}{\pi}}$ *or* $\sqrt[3]{\frac{6V}{\pi}}$

$r = d \div 2$ *or* $\frac{1}{2}\sqrt{\frac{A}{\pi}}$ *or* $\sqrt[3]{\frac{3V}{4\pi}}$

Cylinder



r = **r**adius

d = **d**iameter

h = **h**eight

C = **c**urved area *(without ends)*

T = **t**otal area *(with ends)*

V = **v**olume

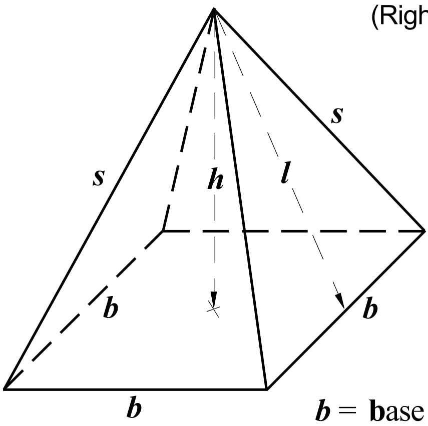
$V = \pi \times r^2 \times h$ *or* $\pi \times d^2 \times h \div 4$ *or* $\frac{C \times r}{2}$

¡ $\frac{2 \times V}{r}$

$T = 2 \times \pi \times r \times (r + h)$

Pyramid

(Right square-based)



b = **b**ase edge length

s = **s**lant edge length

h = perpendicular **h**eight

l = s***l***ant height

V = **v**olume

$V = \pi \times b^2 \times h \div 3$

$h = 3 \times V \div b^2$

$b = \sqrt{\frac{3V}{h}}$

$s = \sqrt{h^2 + \frac{b^2}{2}}$

$b = \sqrt{2(s^2 - h^2)}$

$h = \sqrt{s^2 - \frac{b^2}{2}}$

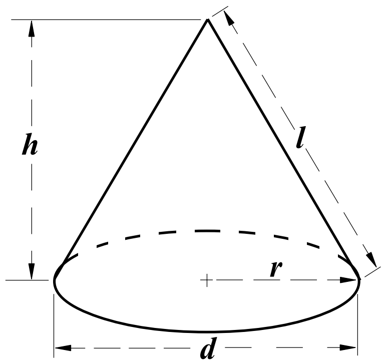
$b = 2\sqrt{(l^2 - h^2)}$

$l = \sqrt{h^2 + \frac{b^2}{4}}$

$h = \sqrt{l^2 - \frac{b^2}{4}}$

Cone

(Right circular)



r = **r**adius of base circle

d = **d**iameter of base

h = perpendicular **h**eight

l = s***l***ant height

C = **c**urved area *(without base)*

V = **v**olume

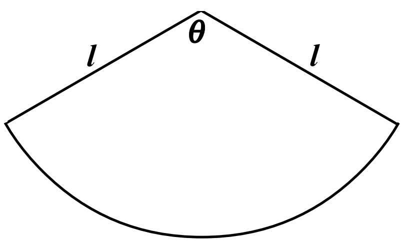
$r = d \div 2$

$l = \sqrt{r^2 + h^2}$ $h = \sqrt{l^2 - r^2}$ $r = \sqrt{l^2 - h^2}$

$V = \pi \times r^2 \times h \div 3$ *or* $\pi \times d^2 \times h \div 12$

$C = \pi \times r \times l$ $r = \sqrt{\frac{3V}{\pi h}}$ $h = \frac{3V}{\pi r^2}$

The sector needed to make a cone having a base radius of ***r*** and slant height of ***l*** can be cut from a circle with a radius of ***l*** and a sector angle of $\theta°$ where $\theta° = \frac{360\,r}{l}$



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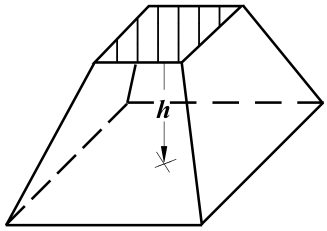
Frustum of a Pyramid



A, B = areas of the top and bottom parallel face

h = perpendicular **h**eight between the two faces

V = **V**olume

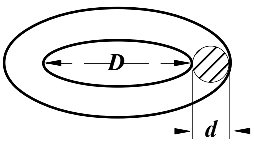
$$V = (A + B + \sqrt{A \times B}) \times h \div 3$$

Note that the shapes of the top and bottom faces do not have to be squares, but they must be the same and parallel to each other.

This formula also applies to the frustum of a cone.

Torus

A torus is a solid circular ring, made of material which has a circular cross-section. It is also known as an **anchor ring**.



D = **D**iameter of inside of torus

d = **d**iameter of circular cross-section

S = **S**urface area

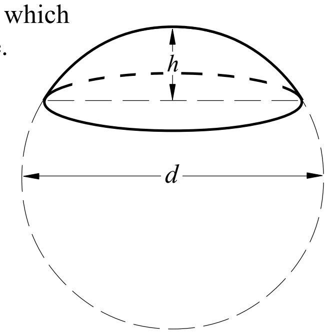
V = **V**olume

$$V = \pi^2 d^2 (D + d) \div 4$$

$$S = \pi^2 d (D + d)$$

Segment of a Sphere

A segment of a sphere is the shape cut off by a single plane which passes through the sphere. It is also known as a **cap**.



d = **d**iameter of sphere

h = **h**eight of segment

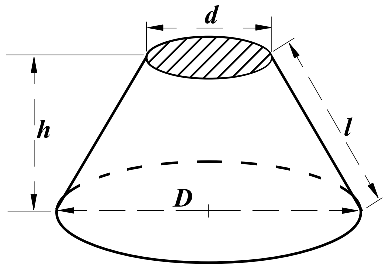
V = **V**olume of segment

A = **A**rea of curved surface of segment

$$V = \pi h^2 (3d - 2h) \div 6$$

$$A = \pi d h$$

Frustum of a Cone



D = **D**iameter of base circle

d = **d**iameter of top circle

h = perpendicular **h**eight between the two faces

l = s**l**ant height

C = **C**urved surface area

V = **V**olume

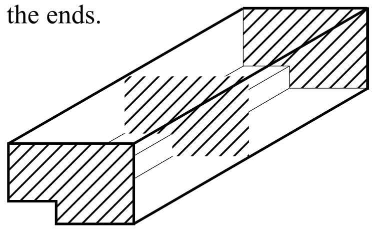
$$V = \pi h (D^2 + Dd + d^2) \div 12$$

$$C = \pi l (D + d) \div 2$$

$$l = \left(\sqrt{(D - d)^2 + 4h^2}\right) \div 2$$

Prisms

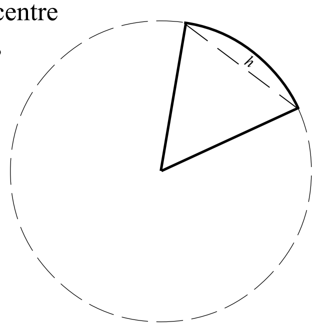
The two end faces of a prism are parallel to each other and the same shape and size. A cross-section anywhere in its length, parallel to the end faces is identical to the ends.



Its **volume** is equal to
The area of one end
× the perpendicular distance between them

Sector of a Sphere

A sector of a sphere is rather like a cone, with its vertex at the centre of the sphere, and the base, instead of being flat, is formed by a segment of the sphere.



d = **d**iameter of sphere

h = **h**eight of base segment

V = **V**olume of sector

$$V = \pi d^2 h \div 6$$

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Identities

$\sin^2 A + \cos^2 A = 1$
$\sec^2 A = 1 + \tan^2 A$
$\text{cosec}^2 A = 1 + \cot^2 A$

Multiple angles

$$\sin 2A = 2\sin A\cos A = \frac{2\tan A}{1+\tan^2 A}$$

$$\cos 2A = \cos^2 A - \sin^2 A = 1 - 2\sin^2 A$$
$$= 2\cos^2 A - 1 = \frac{1-\tan^2 A}{1+\tan^2 A}$$

$$\tan 2A = \frac{2\tan A}{1-\tan^2 A}$$

$$\sin 3A = 3\sin A - 4\sin^3 A$$
$$\cos 3A = 4\cos^3 A - 3\cos A$$
$$\tan 3A = \frac{3\tan A - \tan^3 A}{1 - 3\tan^2 A}$$

Addition & product formulas

$\sin(A+B) = \sin A\cos B + \cos A\sin B$
$\cos(A+B) = \cos A\cos B - \sin A\sin B$
$\sin(A-B) = \sin A\cos B - \cos A\sin B$
$\cos(A-B) = \cos A\cos B + \sin A\sin B$

$$\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A\tan B}$$
$$\tan(A-B) = \frac{\tan A - \tan B}{1 + \tan A\tan B}$$

$$\sin A + \sin B = 2\sin\frac{A+B}{2}\cos\frac{A-B}{2}$$
$$\sin A - \sin B = 2\cos\frac{A+B}{2}\sin\frac{A-B}{2}$$
$$\cos A + \cos B = 2\sin\frac{A+B}{2}\cos\frac{A-B}{2}$$
$$\cos B - \cos A = 2\sin\frac{A+B}{2}\cos\frac{A-B}{2}$$

$\sin(A+B) + \sin(A-B) = 2\sin A\cos B$
$\sin(A+B) - \sin(A-B) = 2\cos A\sin B$
$\cos(A+B) + \cos(A-B) = 2\cos A\cos B$
$\cos(A-B) - \cos(A+B) = 2\sin A\sin B$

Half-angle formulas

$\cos A = \cos^2(\tfrac{A}{2}) - \sin^2(\tfrac{A}{2})$
$\sin A = 2\sin(\tfrac{A}{2})\cos(\tfrac{A}{2})$

$$\tan A = \frac{2\tan(\tfrac{A}{2})}{1-\tan^2(\tfrac{A}{2})}$$

$1 + \cos A = 2\cos^2(\tfrac{A}{2})$
$1 - \cos A = 2\sin^2(\tfrac{A}{2})$

If $t = \tan(\tfrac{A}{2})$ then

$$\sin A = \frac{2t}{1+t^2} \qquad \cos A = \frac{1-t^2}{1+t^2} \qquad \tan A = \frac{2t}{1-t^2}$$

1 radian = 180 ÷ π **degrees**
1 rad ≈ 57.295 779 513 082 320 876 …°

A =	0°	30°	45°	60°	90°
in rads =	0	$\pi/6$	$\pi/4$	$\pi/3$	$\pi/2$
$\sin A$ =	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1
$\cos A$ =	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0
$\tan A$ =	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	∞

Angles in any quadrant

sine is positive (2)	**ALL** are positive (1)
tangent is positive (3)	**cosine** is positive (4)

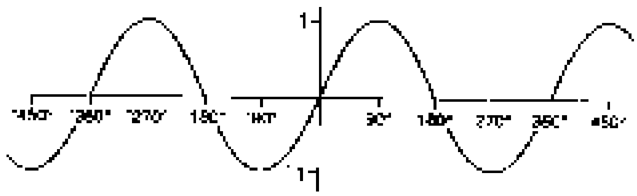
Negative angles

$\sin(-A) = -\sin A$
$\cos(-A) = \cos A$
$\tan(-A) = -\tan A$

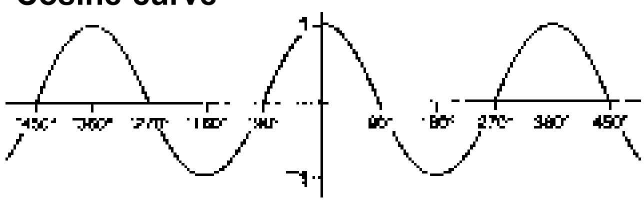
The general angle

If $\sin A = \sin\alpha$ then $A = 180n + (-1)^n\alpha$
If $\cos A = \cos\alpha$ then $A = 360n \pm \alpha$
If $\tan A = \tan\alpha$ then $A = 180n + \alpha$

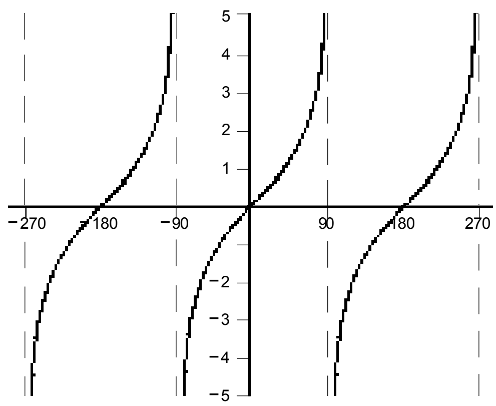
Sine curve



Cosine curve



Tangent curves



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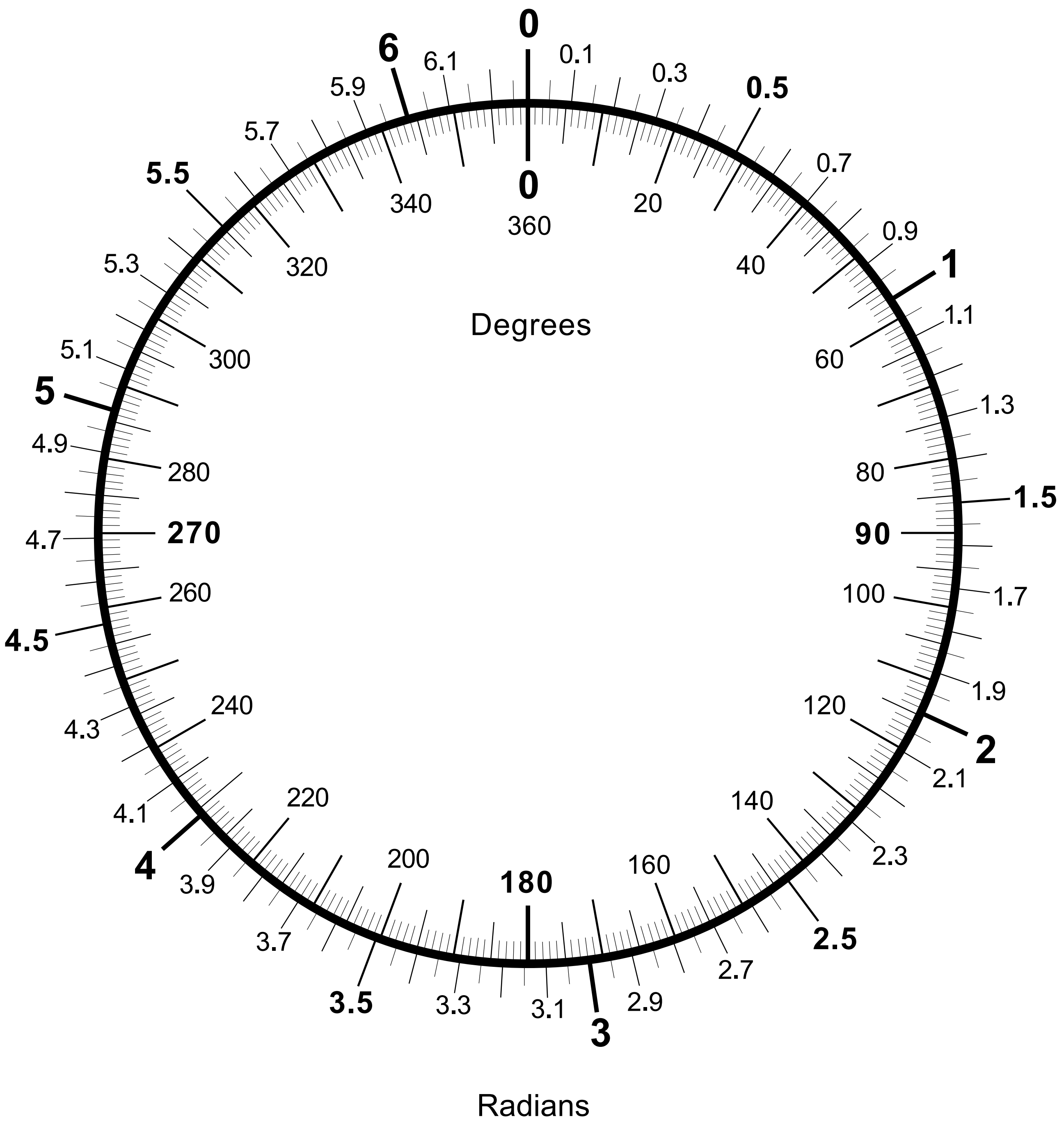
The equivalent values of
degrees
&
radians



1 **radian** = 180 ÷ π **degrees**
1 **rad** ≈ 57.295 779 513 082 320 876 798 154 814 105 170 332 405 …°
There are 2π (≈ 6.283 185 …) **radians** in 360°
1° ≈ 0.017 453 292 519 943 … radians
90° ≈ 1.570 796 326 794 896 619 … radians

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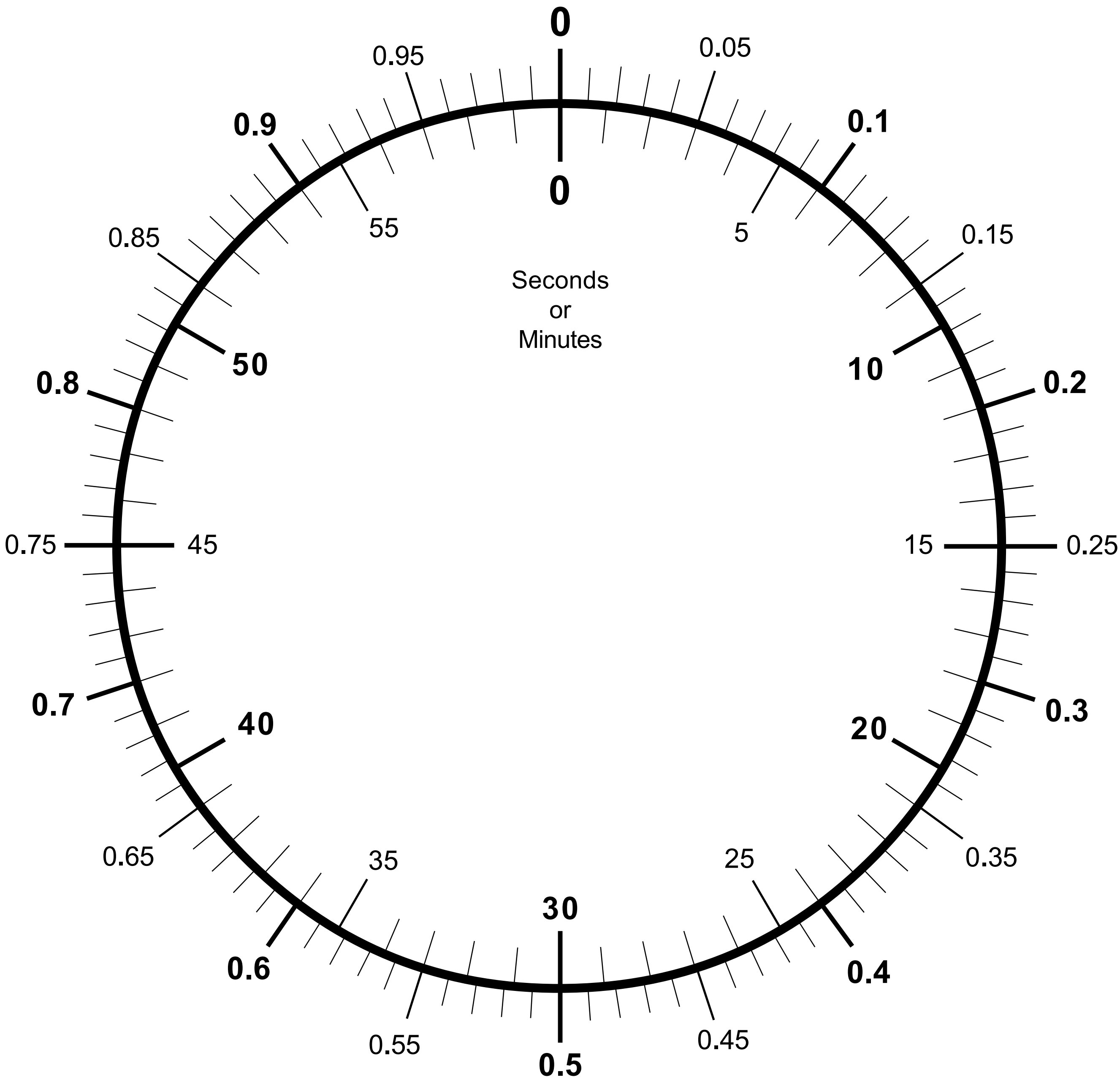
The equivalent values of
seconds or minutes
(of time or angle)
&
a decimal fraction of a
minute, hour or degree



Decimal Fraction of Minute, Hour or Degree

Time		**Angle**	
60 seconds =	1 minute	60 seconds =	1 minute
60 minutes =	1 hour	60 minutes =	1 degree

Time is written in the form hh**:**mm**:**ss *example* 12:34:06
Angle is written in the form d° m' s'' *example* 123° 4' 56''

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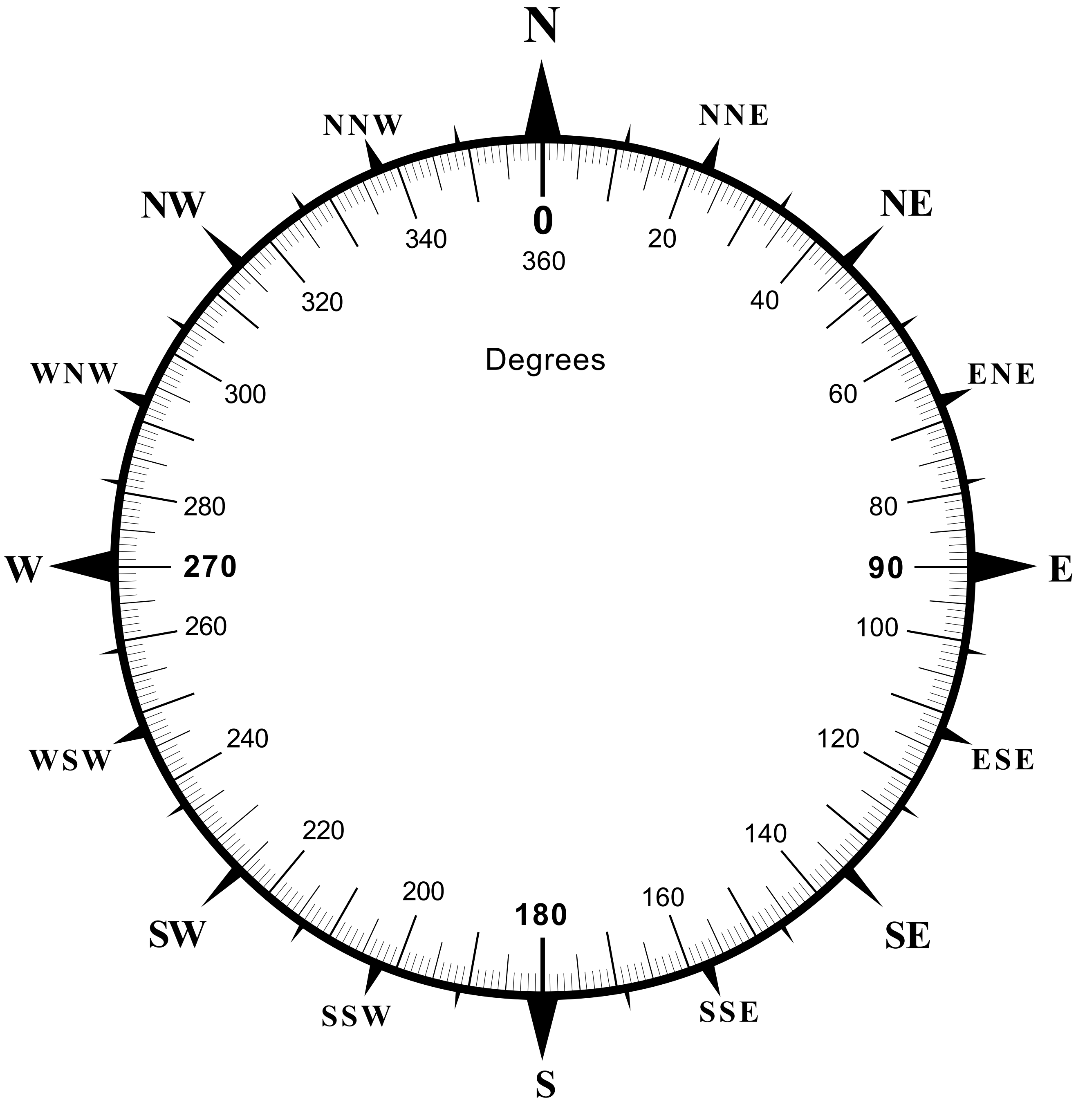
The equivalent values of
degrees
&
the points of the compass



The four familiar compass directions (North, East, South, West) originated in the 1300's though their names were different then. Naming the directions between the four main ones (North-east, South-southwest, etc.) was also done, creating the 16 points named above, and these generally made up what is known as a "Compass-rose" in reference to their usual flower-like appearance.

However, this meant that directions could only be given in (360 ÷ 16 =) 22.5° intervals which is rather large for navigational purposes. So further sub-divisions were made using directions like "Northeast by east" which indicated a direction midway between NE and ENE, producing a total of 32 directions. These are shown, but not named, in the above diagram. The intervals between these were then further divided into quarter-points so that direction intervals of a little under 3° were possible.

Nowadays, all directions are given in degrees.

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Quadratic Equations

If $ax^2 + bx + c = 0$ then

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

If $b^2 - 4ac > 0$ there are two, real, different roots.
If $b^2 - 4ac = 0$ there is only one root.
If $b^2 - 4ac < 0$ the roots are complex.

Indices

$$a^m \times a^n = a^{m+n}$$
$$a^m \div a^n = a^{m-n}$$
$$(a^m)^n = a^{m \times n}$$
$$\sqrt[n]{a^m} = a^{m \div n}$$
$$\sqrt[n]{a} = a^{\frac{1}{n}}$$
$$a^{-n} = \frac{1}{a^n}$$
$$a^0 = 1$$
$$(a \times b)^n = a^n \times b^n$$
$$(a \div b)^n = a^n \div b^n$$

Expansions & Factorisations

$$(a+b)^2 = a^2 + 2ab + b^2$$
$$(a-b)^2 = a^2 - 2ab + b^2$$
$$a^2 - b^2 = (a+b)(a-b)$$
$$(a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$$
$$(a+b)^3 = a^3 + b^3 + 3ab\,(a+b)$$
$$(a-b)^3 = a^3 - 3a^2b + 3ab^2 - b^3$$
$$(a-b)^3 = a^3 - b^3 - 3ab\,(a-b)$$
$$a^3 + b^3 = (a+b)(a^2 - ab + b^2)$$
$$a^3 - b^3 = (a-b)(a^2 + ab + b^2)$$
$$a^4 - b^4 = (a+b)(a^3 - a^2b + ab^2 - b^3)$$
$$a^4 - b^4 = (a-b)(a^3 + a^2b + ab^2 + b^3)$$

$a^n + y^n$ is divisible by $(x+y)$ when n is odd
but by $(x-y)$ never
$a^n - y^n$ is divisible by $(x+y)$ when n is even
and by $(x-y)$ always

Logarithms

If $N = a^x$ then $\log_a N = x$ and $N = a^{\log_a N}$

$$\log(a \times b) = \log a + \log b$$
$$\log(a \div b) = \log a - \log b$$
$$\log a^n = n \log a$$
$$\log \sqrt[n]{a} = \frac{1}{n}\log a$$
$$\log_a N = \frac{\log_b N}{\log_b a}$$
$$\log_e N = 2.3026 \times \log_{10} N$$
$$\log_a 1 = 0$$

Arithmetic Progressions

The general form of an AP is
$a,\ a+d,\ a+2d,\ a+3d,\ a+4d,\ \ldots\ldots\ a+(n-1)d$
where
a = the first term
d = the common difference
n = the number of terms
the last term is
$l = a + (n - l)d$
the total sum of n terms is
$S_n = n(a+1) \div 2$ *or* $n[2a + (n-1)d] \div 2$

Geometic Progressions

The general form of a GP is
$a,\ ar,\ ar^2,\ ar^3,\ ar^4,\ ar^5,\ \ldots\ldots\ ar^{n-1}$
where
a = the first term
r = the common ratio *or* multiplier
n = the number of terms
the total sum of n terms is
$S_n = a(1 - r^n) \div (1 - r)$ if $r < 1$
or
$S_n = a(r^n - 1) \div (r - 1)$ if $r > 1$
if n is infinity and $r^2 < 1$ then
$S_\infty = a \div (1 - r)$
The geometric mean of two numbers a and b = $\sqrt{ab}$

Sums of Powers of Natural Numbers

The first n natural numbers are
$1,\ 2,\ 3,\ 4,\ 5,\ 6,\ 7,\ \ldots\ldots\ n$
Their sum when each has been raised to the power r is
$\Sigma n^r = 1^r + 2^r + 3^r + 4^r + 5^r + 6^r + \ldots\ldots + n^r$
For any given value of r there is a formula for Σn^r
The first six are

$(r=1)$ $\Sigma n = n(n+1) \div 2$
$(r=2)$ $\Sigma n^2 = n(n+1)(2n+1) \div 6$
$(r=3)$ $\Sigma n^3 = n^2(n+1)^2 \div 4$ *or* $(\Sigma n)^2$
$(r=4)$ $\Sigma n^4 = n(n+1)(2n+1)(3n^2+3n-1) \div 30$
$(r=5)$ $\Sigma n^5 = n^2(n+1)^2(2n^2+2n-1) \div 12$
$(r=6)$ $\Sigma n^6 = n(n+1)(2n+1)(3n^4+6n^3-3n+1) \div 42$

Combinations

Given ***n*** different objects and required to choose ***r*** at a time, this formula gives the number of ways in which it can be done, neglecting the order in which they are chosen.

$${}^n\mathrm{C}_r = \frac{n!}{(n-r)!\,r!}$$

Given the importance of these numbers in the Binomial Theorem below, they are also known as the **Binomial Coefficients**. (*see Table of Values at the back*)

Binomial Theorem

$$(a+b)^n = a^n + {}^n\mathrm{C}_1 a^{n-1}b + {}^n\mathrm{C}_2 a^{n-2}b^2 + {}^n\mathrm{C}_3 a^{n-3}b^3 + \ldots$$
$$\ldots + {}^n\mathrm{C}_r a^{n-r}b^r + \ldots \ldots + b^n$$

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Trigonometric (x in radians)

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \frac{x^9}{9!} - \ldots$$

$$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \frac{x^8}{8!} - \ldots$$

$$\tan x = x + \frac{2x^3}{3!} + \frac{16x^5}{5!} + \frac{272x^7}{7!} + \frac{7936x^9}{9!} + \ldots$$

$$\sin^{-1} x = x + \frac{1}{2}\frac{x^3}{3} + \frac{1\times 3}{2\times 4}\frac{x^5}{5} + \frac{1\times 3\times 5}{2\times 4\times 6}\frac{x^7}{7} + \ldots \quad (-1 \leqslant x \leqslant 1)$$

$$\tan^{-1} x = x - \frac{x^3}{3} + \frac{x^5}{5} - \frac{x^7}{7} + \frac{x^9}{9} - \ldots \quad (-\pi/2 < x < \pi/2)$$

Hyperbolic

$$\sinh x = x + \frac{x^3}{3!} + \frac{x^5}{5!} + \frac{x^7}{7!} + \frac{x^9}{9!} + \ldots \qquad \sinh x = \frac{e^x - e^{-x}}{2}$$

$$\cosh x = 1 + \frac{x^2}{2!} + \frac{x^4}{4!} + \frac{x^6}{6!} + \frac{x^8}{8!} + \ldots \qquad \cosh x = \frac{e^x + e^{-x}}{2}$$

$$\tanh x = x \frac{2x^3}{3!} - \frac{16x^5}{5!} + \frac{272x^7}{7!} - \frac{7936x^9}{9!} + \ldots \qquad \tanh x = \frac{e^x - e^{-x}}{e^x + e^{-x}}$$

Taylor's expansion

$$f(x+a) = f(x) + af'(x) + \frac{a^2}{2!}f''(x) + \frac{a^3}{3!}f'''(x) + \ldots$$

Maclaurin's expansion

$$f(x) = f(0) + xf'(0) + \frac{x^2}{2!}f''(0) + \frac{x^3}{3!}f'''(0) + \ldots$$

Logarithmic

$$\log_e(1+x) = x - \frac{x^2}{2!} + \frac{x^3}{3!} - \frac{x^4}{4!} + \ldots \quad (\text{for } -1 < x \leqslant 1)$$

$$\log_e(1-x) = -x - \frac{x^2}{2!} - \frac{x^3}{3!} - \frac{x^4}{4!} - \ldots \quad (\text{for } -1 \leqslant x < 1)$$

Exponential

$$e = 1 + \frac{1}{1!} + \frac{1}{2!} + \frac{1}{3!} + \frac{1}{4!} + \ldots$$

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \ldots$$

Pi

$$\frac{\pi}{4} = 1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \frac{1}{9} - \ldots$$

$$\frac{\pi^2}{8} = \frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \frac{1}{7^2} + \frac{1}{9^2} + \ldots$$

$$\frac{\pi^3}{32} = \frac{1}{1^3} - \frac{1}{3^3} + \frac{1}{5^3} - \frac{1}{7^3} + \frac{1}{9^3} - \ldots$$

$$\frac{\pi}{2} = \frac{2}{1} \times \frac{2}{3} \times \frac{4}{3} \times \frac{4}{5} \times \frac{6}{5} \times \frac{6}{7} \times \frac{8}{7} \times \ldots$$

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function $f(x)$ or $y = f(x)$	(1st) derivative $f'(x)$ or $\frac{d}{dx}f(x)$ or $\frac{dy}{dx}$	integral $\int f(x)\,dx$ or $\int y\,dx$
x^n	nx^{n-1}	$\frac{1}{n+1}x^{n+1}$
e^x	e^x	e^x
e^{ax}	ae^{ax}	$\frac{1}{a}e^{ax}$
a^x	$a^x \log_e a$	$\frac{1}{\log_e a}a^x$
$\log_e x$	$\frac{1}{x}$	$x\log_e x - x$
$\frac{1}{x}$	$-\frac{1}{x^2}$	$\log_e\|x\|$
$\frac{1}{a^2+x^2}$		$\frac{1}{a}\tan^{-1}\frac{x}{a}$
$\frac{1}{a^2-x^2}$		$\frac{1}{a}\tanh^{-1}\frac{x}{a}$
$\frac{1}{x^2-a^2}$		$-\frac{1}{a}\coth^{-1}\frac{x}{a}$
$\frac{1}{\sqrt{a^2+x^2}}$		$\sinh^{-1}\frac{x}{a}$
$\frac{1}{\sqrt{x^2-a^2}}$		$\cosh^{-1}\frac{x}{a}$
$\sin x$	$\cos x$	$-\cos x$
$\cos x$	$-\sin x$	$\sin x$
$\tan x$	$\sec^2 x$	$\log_e\|\sec x\|$
$\sin^{-1} x$	$\frac{1}{\sqrt{1-x^2}}$	
$\cos^{-1} x$	$-\frac{1}{\sqrt{1-x^2}}$	
$\tan^{-1} x$	$\frac{1}{1+x^2}$	

constants of integration
have not been shown

Given that u and v are both functions of x

Product rule

if $y = u \times v$ then $\frac{dy}{dx} = v\frac{du}{dx} + u\frac{dv}{dx}$

Quotient rule

if $y = u \div v$ then $\frac{dy}{dx} = \left(v\frac{du}{dx} - u\frac{dv}{dx}\right) \div v^2$

Chain rule

if y is a function of u then $\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$

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In statistics, when the data content is numerical, it is usual to use the symbol x to represent the general case, and individual pieces of data as $x_1\ x_2\ x_3\ x_4\ x_5\ x_6\ x_7\ \dots\ x_n$
Another commonly used symbol is **Σ** *(Greek sigma)* which means "find the sum of". So a formula containing Σx would mean "add up all the x-numbers", and Σx^2 would mean "square all the x-numbers and add up all those values". The number of pieces of data is given by n.
If the data is grouped, then f is used to refer to the frequency of the data in each group and that would require a change to some of the formulas given here.

Arithmetic Mean

Generally this is referred to simply as the mean.

Symbol is $\bar{x}$

This may be found by

Adding up the values of all the data
Dividing by the number of pieces of data

Expressed as a formula it is $\bar{x} = \dfrac{\sum x}{n}$

Range

is the absolute value of the difference between the greatest and least values of the data.
Expressed as a formula it is

range $= |x_{max} - x_{min}|$

Root Mean Square Value

is given by $\sqrt{\dfrac{\sum x^2}{n}}$

Standard Deviation

This may be found by

Squaring the values of all the data
Adding them all up
Dividing by how many there are
Subtracting the square of the mean value
Taking the square root.

Symbol is σ

Expressed as a formula it is $\sigma = \sqrt{\dfrac{\sum x^2}{n} - \bar{x}^2}$

Variance

is the square of the Standard Deviation

$= \sigma^2$

χ^2 (chi-squared) Test

For any particular piece of data, if

O is its **O**bserved frequency
and
E is its **E**xpected frequency

then

$$\chi^2 = \sum \frac{(O-E)^2}{E}$$

which is the summation carried out over all the groups of the data

Correlation Coefficient

More precisely it is

Pearson's product moment correlation coefficient
Symbol is r

When the data is in the form of ordered pairs of numbers such as (x, y) and there are n such pairs, then the amount of correlation between them can be determined by

A. Multiplying the matching x and y values together, adding them all up and multiplying the total by n
B. Adding up all x-values; adding up all y-values; and multiplying the two results together.
C. Subtracting the result of **B** from **A** *(It might be negative)*
D. Squaring all x-values, adding them up, multiplying the total by n. Repeating for y-values.
E. Adding together all x-values, and squaring the total. Repeating for y-values.
F. Subtracting the x-result in **E** from that in **D** and repeating that for y-result.
G. Multiplying the two answers from **F** together and taking the square root.

Then r = result from **C** ÷ result from **G**

Expressed as a formula it is

$$r = \frac{n\sum xy - \sum x \sum y}{\sqrt{\left[n\sum x^2 - \left(\sum x\right)^2\right]\left[n\sum y^2 - \left(\sum y\right)^2\right]}}$$

Straight Line Formula

When the data is in the form of ordered pairs of numbers such as (x, y) and there is a good degree of correlation between them *(as determined above)* then it is possible, as well as useful, to draw a straight line which can serve as the basis of further calculations.
The equation for this line will be of the form

$$y = \mathbf{m}x + \mathbf{c}$$

The necessary values of 'm' and 'c' can be found from

$$\mathbf{m} = \frac{n\sum xy - \sum x \sum y}{n\sum x^2 - \left(\sum x\right)^2}$$

and

$$\mathbf{c} = \frac{\sum y - \mathrm{m}\sum x}{n}$$

Rank Order Correlation Coefficient

More precisely it is

Spearman's rank order correlation coefficient
Symbol is ρ

When two sets of data have been ranked in order by some criteria or other, this coefficient is used to determine how closely the two lists agree (or differ).
Given that there are n items listed, it is found by

Finding the difference in value (by their list order) of each corresponding pair of rankings.
Squaring all the differences.
Adding the squared values together and multiplying by 6
Dividing the previous result by $(n^3 - n)$
Subtracting that from 1

$= \rho$

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Systems of Measurement and their Units

There are three main systems of measurement still in use. These are known as: **imperial units** shown here as (UK); U.S. American units shown as (US); and **metric units** which form the basis of the Système Internationale or SI. The principal units in each of those systems (with the defining measure printed in **bold**) and their relationship are

Length (UK) & (US)

12	inches	≡	1	foot
3	feet	≡	1	**yard**
22	yards	≡	1	chain
10	chains	≡	1	furlong
8	furlongs	≡	1	mile
1760	yards	≡	1	mile
5280	feet	≡	1	mile

Length (metric)

10	millimetres	≡	1	centimetre (cm)
10	centimetres	≡	1	decimetre (dm)
10	decimetres	≡	1	**metre (m)**
10	metres	≡	1	decametre (dam)
10	decametres	≡	1	hectometre (hm)
10	hectometres	≡	1	kilometre (km)

Area (UK) & (US)

144	square inches	≡	1	square foot
9	square feet	≡	1	square yard
4840	square yards	≡	1	acre
640	acres	≡	1	square mile

Area (metric)

100	square mm	≡	1	square cm (cm^2)
100	square cm	≡	1	square dm (dm^2)
100	square dm	≡	1	square metre (m^2)
100	square metres	≡	1	are (a)
100	ares	≡	1	hectare (ha)
100	hectares	≡	1	square km (km^2)
10000	square cm	≡	1	square metre (m^2)
1 million	square metres	≡	1	square km (km^2)

Volume (UK) & (US)

1728	cubic inches	≡	1	cubic foot
27	cubic feet	≡	1	cubic yard

Volume (metric)

1000	cubic mm	≡	1	cubic cm (cm^3)
1000	cubic cm	≡	1	cubic dm (dm^3)
1000	cubic dm	≡	1	cubic metre (m^3)

Mass (UK) & (US)

16	ounces	≡	1	**pound**
2000	pounds	≡	1	short ton (US)
2240	pounds	≡	1	long ton (UK)
437.5	grains	≡	1	ounce
14	pounds	≡	1	stone
8	stones	≡	1	hundredweight (UK)
20	hundredweight	≡	1	ton (UK)

Mass (metric)

1000	milligrams (mg)	≡	1	gram (g)
1000	grams	≡	1	**kilogram (kg)**
1000	kilograms	≡	1	tonne (t)
200	milligrams	≡	1	carat

Capacity (UK) & (US liquid)

4	gills	≡	1	pint
2	pints	≡	1	quart
4	quarts	≡	1	**gallon**

Capacity (US dry)

2	pints	≡	1	quart
8	quarts	≡	1	peck
4	pecks	≡	1	**bushel**

Note (UK) and (US) gallons are NOT the same size so other measures of capacity having the same name are NOT the same size. Also a liquid pint is NOT the same size as a dry pint.

Capacity (metric)

10	millilitres (ml)	≡	1	centilitre (cl)
1000	millilitres	≡	1	**litre (1 or L)**
1000	litres	≡	1	cubic metre

Originally every system had its own standard on which the other measures were based. Now the SI standards are accepted worldwide and all other measures are based on that. These values are **exact** -

1 yard ≡ 0.9144 metres

1 pound ≡ 0.453 592 37 kilograms

1 gallon (UK) ≡ 4.546 09 litres

1 gallon (US liquid) ≡ 3.785 411 784 litres

1 bushel (US) ≡ 35.239 070 166 88 litres

The old measure of a **troy ounce** is still used for dealing in precious metals and stones.

1 troy ounce ≡ 31.103 476 8 grams

Another measure of a **barrel** is often used for dealing in crude oil 1 barrel ≡ 42 gallons (US)

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The SI defines seven base units.

Four of these with their abbreviations (in brackets) and definitions are -

metre (m) The metre is the unit of **length**. It is the distance light travels, in a vacuum, in $\frac{1}{299\,792\,458}$ th of a second.

kilogram (kg) The kilogram is the unit of **mass**. It is the mass of an international prototype in the form of a platinum-iridium cylinder kept at Sèvres in France. *It is now the only base unit still defined in terms of a material object, and also the only one with a prefix (kilo-) already in place.*

second (s) The second is the unit of **time**. It is the time taken for 9 192 631 770 periods of vibration of the caesium-133 atom to occur.

kelvin (K) The kelvin is the unit of thermodynamic **temperature**. It is $\frac{1}{273.16}$ th of the thermodynamic temperature of the triple point of water.

The other three are:

ampere (A) for measuring current;

mole (mol) for measuring amounts of a substance;

candela (cd) for measuring the intensity of light.

All other units are derived from those seven.

There are rules for using the SI. Some of the more important are -

- A unit may take only one prefix. (µg *not* mmg)
- There should be no space between a prefix and the unit to which it belongs (nm *not* n m)
- A combination of two (or more) units should have either a space or a half-high dot between them (N m *or* N•m *but not* Nm)
- To make numbers easier to read they may be divided into groups of three separated by spaces (or half-spaces) but NOT commas. (403 178 *not* 403,178)
- Whole numbers should be separated from their decimal part by a comma. A point is acceptable but it must be placed on the line of the bottom edge of the number and not in a mid-way position. (2,3 *or* 2.3 *but not* 2·3)
- There should be a space between a number and the unit (3 m *not* 3m)
- Symbols for units are not pluralised (4 metres or 4 m *not* 4 ms)

The Decimal Metric System came into existence during the time of the French Revolution at the end of the 1700's, and was gradually adopted, principally by scientists, in the 1800's.

This led (in 1874) to the British Association for the Advancement of Science introducing the first coherent system based on metric units, using the centimetre, gram, second (the **cgs** system). This was followed (in 1889) by a second system based on the metre, kilogram, second (the **MKS** system). Both of these systems were used for many years, and it was not until the middle of the 1900's that the MKS became the standard in school texts.

The International System of Units (usually identified as the **SI** system) officially came into being as the Système International d'Unités at the 11th General Conference of Weights and Measures held in Paris in October 1960. Generally speaking it was a formalisation of the MKS system.

By the late 1900's the SI system was in general use among scientists (of all branches), engineers, manufacturers, most trades, and those engaed in international commerce. There are a few exceptions, especially in commodities, where older units are still retained, but these are specialised usages and will undoubtedly disappear in time.

The work of metricating public usage, both nationally and internationally, was also started. Principally this means only the metre and kilogram (with their derivatives). The signs are that it will be several decades before this process will be be completed.

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The **SI** allows units of different sizes to be created from the standard ones by using prefixes, which act as multipliers. This list gives all the prefixes allowed in the **SI**, the single letter or symbol to be used in the abbreviated form, and the multiplying factor they represent, both in index notation and in full. Note the difference between using capital letters and small letters.

yotta	**Y**	$\times 10^{24}$	1 000 000 000 000 000 000 000 000
zetta	**Z**	$\times 10^{21}$	1 000 000 000 000 000 000 000
exa	**E**	$\times 10^{18}$	1 000 000 000 000 000 000
peta	**P**	$\times 10^{15}$	1 000 000 000 000 000
tera	**T**	$\times 10^{12}$	1 000 000 000 000
giga	**G**	$\times 10^{9}$	1 000 000 000
mega	**M**	$\times 10^{6}$	1 000 000
kilo	**k**	$\times 10^{3}$	1 000
hecto	**h**	$\times 10^{2}$	100
deca	**da**	$\times 10^{1}$	10
		10^{0}	1
deci	**d**	$\times 10^{-1}$	0.1
centi	**c**	$\times 10^{-2}$	0.01
milli	**m**	$\times 10^{-3}$	0.001
micro	**µ**	$\times 10^{-6}$	0.000 001
nano	**n**	$\times 10^{-9}$	0.000 000 001
pico	**p**	$\times 10^{-12}$	0.000 000 000 001
femto	**f**	$\times 10^{-15}$	0.000 000 000 000 001
atto	**a**	$\times 10^{-18}$	0.000 000 000 000 000 001
zepto	**z**	$\times 10^{-21}$	0.000 000 000 000 000 000 001
yocto	**y**	$\times 10^{-24}$	0.000 000 000 000 000 000 000 001

'**deka**' and/or '**dk**' are used in some places instead of 'deca' and 'da'.

In the early days of computing it was noticed that 2^{10} was very close in value to 1000 and so the practice was adopted of using 'kilo' to mean 1000 in cases like 'kilobyte' and so on up the scale. (This has even spread to money, in reference to salaries things like £35k are seen.)
But, computer usage is based on a binary scale and 2^{10} bytes = 1024 bytes. That difference is small, but it increases as the prefixes get bigger as can be seen in this table.

Name	SI value	Computer value	% difference
kilo	$(10^3)^1$ = 1000	$(2^{10})^1$ = 1024	2.4
mega	$(10^3)^2$ = 1000 000	$(2^{10})^2$ = 1048 576	4.9
giga	$(10^3)^3$ = 1000 000 000	$(2^{10})^3$ = 1073 741 824	7.4
tera	$(10^3)^4$ = 1000 000 000 000	$(2^{10})^4$ = 1099 511 627 776	10
	and so on …	and so on …	

These differences may seem trivial in 'ordinary' contexts but become very important in the telecommunications industry where the higher values are much used and some precision is needed. So, in 1999/2000 the International Electrotechnical Commission [IEC] published a new series of prefixes to be used alongside the SI prefixes and avoid all ambiguity.
These prefixes are made by combining the first two letters of the SI prefix with the first two letters of the word 'binary' so

$(2^{10})^1$ = **ki**lo + **bi**nary becomes **kibi** with the abbreviation **Ki** (note capital K)
$(2^{10})^2$ = **me**ga + **bi**nary becomes **mebi** with the abbreviation **Mi**
$(2^{10})^3$ = **gi**ga + **bi**nary becomes **gibi** with the abbreviation **Gi**
$(2^{10})^4$ = **te**ra + **bi**nary becomes **tebi** with the abbreviation **Ti**
$(2^{10})^5$ = **pe**ta + **bi**nary becomes **pebi** with the abbreviation **Pi**
$(2^{10})^6$ = **ex**a + **bi**nary becomes **exbi** with the abbreviation **Ei**

The difference between, say, 'megabytes' [Mbytes] and 'mebibytes' [Mibytes] is now very clear.

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Conversion Factors for Units of Measure

These **conversion factors** are multipliers (or dividers) which can be used to change the numerical value of a measure in one type of unit into its equivalent value when measured in another type of unit.

To change . .	*into . .*	do this . .
acres	*hectares*	× 0.4047
acres	*sq. kilometres*	÷ 247
acres	*sq. metres*	× 4047
acres	*sq. miles*	÷ 640 #
barrels (oil)	*cu. metres*	÷ 6.29
barrels (oil)	*gallons (UK)*	× 34.97
barrels (oil)	*gallons (US)*	× 42 #
barrels (oil)	*litres*	× 159
centimetres	*feet*	÷ 30.48
centimetres	*inches*	÷ 2.54 #
centimetres	*metres*	÷ 100 #
centimetres	*millimetres*	× 10 #
cubic cm	*cu. inches*	× 0.06102
cubic cm	*litres*	÷ 1000 #
cubic cm	*millilitres*	× 1 #
cubic feet	*cubic inches*	× 1728
cubic feet	*cubic metres*	× 0.0283
cubic feet	*cubic yards*	÷ 27 #
cubic feet	*gallons (UK)*	× 6.229
cubic feet	*gallons (US)*	× 7.481
cubic feet	*litres*	× 28.32
cubic inches	*cu. cm*	× 16.39
cubic inches	*litres*	× 0.01639
cubic metres	*cubic feet*	× 35.31
feet	*cm*	× 30.48 #
feet	*metres*	× 0.3048 #
feet	*yards*	÷ 9 #
fluid ounces (UK)	*fluid ounces (US)*	× 0.961
fluid ounces (UK)	*millitres*	× 28.41
fluid ounces (US)	*fluid ounces (UK)*	× 1.041
fluid ounces (US)	*millitres*	× 29.57
gallons	*pints*	× 8 #
gallons (UK)	*cu. feet*	× 0.1605
gallons (UK)	*gallons (US)*	× 1.2009
gallons (UK)	*litres*	× 4.54609 #
gallons (US)	*cu. feet*	× 0.1337
gallons (US)	*gallons (UK)*	× 0.8327
gallons (US)	*litres*	× 3.785
grams	*kilograms*	÷ 1000 #
grams	*ounces*	÷ 28.35
hectares	*acres*	× 2.471
hectares	*sq. km*	÷ 100 #
hectares	*sq. metres*	× 10000 #
hectares	*sq. miles*	÷ 259
hectares	*sq. yards*	× 11 960
inches	*centimetres*	× 2.54 #
inches	*feet*	÷ 12 #
kilograms	*ounces*	× 35.3
kilograms	*pounds*	× 2.2046
kilograms	*tonnes*	÷ 1000 #
kilograms	*tons (long/UK)*	÷ 1016
kilograms	*tons (short/US)*	÷ 907
kilometres	*metres*	× 1000 #
kilometres	*miles*	× 0.6214
litres	*cubic inches*	× 61.02
litres	*gallons (UK)*	× 0.2200
litres	*gallons (US)*	× 0.2642
litres	*pints (UK)*	× 1.760
litres	*pints (US liquid)*	× 2.113
metres	*yards*	÷ 0.9144 #
metres	*centimetres*	× 100 #
miles	*kilometres*	× 1.609
millimetres	*inches*	÷ 25.4 #
ounces	*grams*	× 28.35
pints (UK)	*litres*	× 0.5683
pints (UK)	*pints (US liquid)*	× 1.201
pints (US liquid)	*litres*	× 0.4732
pints (US liquid)	*pints (UK)*	× 0.8327
pounds	*kilograms*	× 0.4536
pounds	*ounces*	× 16 #
square cm	*sq. inches*	× 0.1550
square feet	*sq. inches*	× 144 #
square feet	*sq. metres*	× 0.0929
square inches	*square cm*	× 6.4516 #
square inches	*square feet*	÷ 144 #
square km	*acres*	× 247
square km	*hectares*	× 100 #
square km	*square miles*	× 0.3861
square metres	*acre*	÷ 4047
square metres	*hectares*	÷ 10 000 #
square metres	*sq. feet*	× 10.76
square metres	*sq. yards*	× 1.196
square miles	*acres*	× 640 #
square miles	*hectares*	× 259
square miles	*sq. km*	× 2.590
square yards	*sq. metres*	÷ 1.196
tonnes	*kilograms*	× 1000 #
tonnes	*tons (long/UK)*	× 0.9842
tonnes	*tons (short/US)*	× 1.1023
tons (long/UK)	*kilograms*	× 1016
tons (long/UK)	*tonnes*	× 1.016
tons (short/ US)	*kilograms*	× 907.2
tons (short/US)	*tonnes*	× 0.9072
yards	*metres*	× 0.9144 #

# indicates an **exact** figure, all others are approximations.

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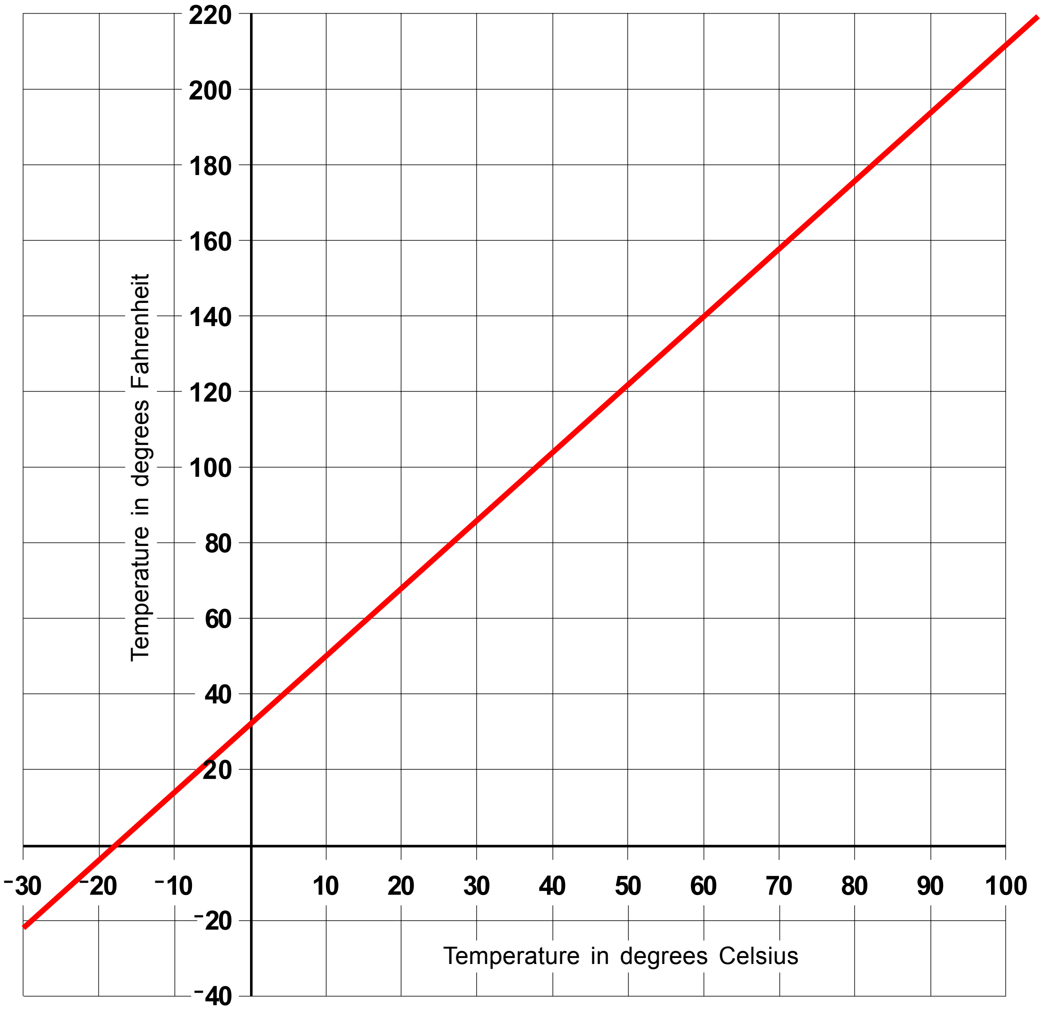

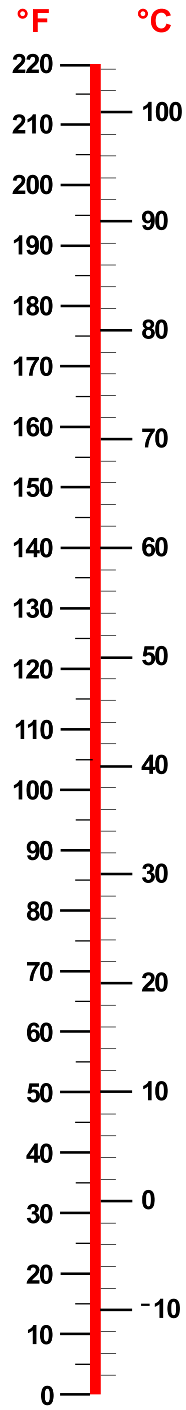

°C ≡ degrees Celsius
°F ≡ degrees Fahrenheit
K ≡ Kelvin

$$°C = \frac{(°F - 32) \times 5}{9}$$

$$°C = K - 273.15$$

$$°F = \frac{°C \times 9}{5} + 32$$

$$°F = K - 459.67$$

$$K = °C + 273.15$$

$$K = °F + 459.67$$

Absolute zero = 0K *or* ⁻273.15°C *or* ⁻459.67°F
0°C ≡ 32°F *or* 273.15K
100°C ≡ 212°F *or* 373.15K
0°F ≡ ⁻17.78°C *or* 255.37K
98.4°F ≡ 36.9°C *or* 310K

Note that temperatures expressed on the Kelvin scale do not have a degree sign

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Values of ^nC_r

$$^nC_r = \frac{n!}{(n-r)!\,r!}$$

$n\downarrow$	$r\rightarrow$ 2	3	4	5	6	7	8	9	10	11	$n\downarrow$
1											**1**
2	1										**2**
3	3	1									**3**
4	6	4	1								**4**
5	10	10	5								**5**
6	15	20	15	6	1						**6**
7	21	35	35	21	7	1					**7**
8	28	56	70	56	28	8	1				**8**
9	36	84	126	126	84	36	9	1			**9**
10	45	120	210	252	210	120	45	10	1		**10**
11	55	165	330	462	462	330	165	55	11	1	**11**
12	66	220	495	792	924	792	495	220	66	12	**12**
13	78	286	715	1 287	1 716	1 716	1 287	715	286	78	**13**
14	91	364	1 001	2 002	3 003	3 432	3 003	2 002	1 001	364	**14**
15	105	455	1 365	3 003	5 005	6 435	6 435	5 005	3 003	1 365	**15**
16	120	560	1 820	4 368	8 008	11 440	12 870	11 440	8 008	4 368	**16**
17	136	680	2 380	6 188	12 376	19 448	24 310	24 310	19 448	12 376	**17**
18	153	816	3 060	8 568	18 564	31 824	43 758	48 620	43 758	31 824	**18**
19	171	969	3 876	11 628	27 132	50 388	75 582	92 378	92 378	75 582	**19**
20	190	1 140	4 845	15 504	38 760	77 520	125 970	167 960	184 756	167 960	**20**
21	210	1 330	5 985	20 349	54 264	116 280	203 490	293 930	352 716	352 716	**21**
22	231	1 540	7 315	26 334	74 613	170 544	319 770	497 420	646 646	705 432	**22**
23	253	1 771	8 855	33 649	100 947	245 157	490 314	817 190	1 144 066	1 352 078	**23**
24	276	2 024	10 626	42 504	134 596	346 104	735 471	1 307 504	1 961 256	2 496 144	**24**
25	300	2 300	12 650	53 130	177 100	480 700	1 081 575	2 042 975	3 268 760	4 457 400	**25**
26	325	2 600	14 950	65 780	230 230	657 800	1 562 275	3 124 550	5 311 735	7 726 160	**26**
27	351	2 925	17 550	80 730	296 010	888 030	2 220 075	4 686 825	8 436 285	13 037 895	**27**
28	378	3 276	20 475	98 280	376 740	1 184 040	3 108 105	6 906 900	13 123 110	21 474 180	**28**
29	406	3 654	23 751	118 755	475 020	1 560 780	4 292 145	10 015 005	20 030 010	34 597 290	**29**
30	435	4 060	27 405	142 506	593 775	2 035 800	5 852 925	14 307 150	30 045 015	54 627 300	**30**
31	465	4 495	31 465	169 911	736 281	2 629 575	7 888 725	20 160 075	44 352 165	84 672 315	**31**
32	496	4 960	35 960	201 376	906 192	3 365 856	10 518 300	28 048 800	64 512 240	129 024 480	**32**
33	528	5 456	40 920	237 336	1 107 568	4 272 048	13 884 156	38 567 100	92 561 040	193 536 720	**33**
34	561	5 984	46 376	278 256	1 344 904	5 379 616	18 156 204	52 451 256	131 128 140	286 097 760	**34**
35	595	6 545	52 360	324 632	1 623 160	6 724 520	23 535 820	70 607 460	183 579 396	417 225 900	**35**
36	630	7 140	58 905	376 992	1 947 792	8 347 680	30 260 340	94 143 280	254 186 856	600 805 296	**36**
37	666	7 770	66 045	435 897	2 324 784	10 295 472	38 608 020	124 403 620	348 330 136	854 992 152	**37**
38	703	8 436	73 815	501 942	2 760 681	12 620 256	48 903 492	163 011 640	472 733 756	1 203 322 288	**38**
39	741	9 139	82 251	575 757	3 262 623	15 380 937	61 523 748	211 915 132	635 745 396	1 676 056 044	**39**
40	780	9 880	91 390	658 008	3 838 380	18 643 560	76 904 685	273 438 880	847 660 528	2 311 801 440	**40**

Prime Numbers

2	233	547	877	1229	1597	1993	2371	2749	3187	3581	4001	4421
3	239	557	881	1231	1601	1997	2377	2753	3191	3583	4003	4423
5	241	563	883	1237	1607	1999	2381	2767	3203	3593	4007	4441
7	251	569	887	1249	1609	2003	2383	2777	3209	3607	4013	4447
11	257	571	907	1259	1613	2011	2389	2789	3217	3613	4019	4451
13	263	577	911	1277	1619	2017	2393	2791	3221	3617	4021	4457
17	269	587	919	1279	1621	2027	2399	2797	3229	3623	4027	4463
19	271	593	929	1283	1627	2029	2411	2801	3251	3631	4049	4481
23	277	599	937	1289	1637	2039	2417	2803	3253	3637	4051	4483
29	281	601	941	1291	1657	2053	2423	2819	3257	3643	4057	4493
31	283	607	947	1297	1663	2063	2437	2833	3259	3659	4073	4507
37	293	613	953	1301	1667	2069	2441	2837	3271	3671	4079	4513
41	307	617	967	1303	1669	2081	2447	2843	3299	3673	4091	4517
43	311	619	971	1307	1693	2083	2459	2851	3301	3677	4093	4519
47	313	631	977	1319	1697	2087	2467	2857	3307	3691	4099	4523
53	317	641	983	1321	1699	2089	2473	2861	3313	3697	4111	4547
59	331	643	991	1327	1709	2099	2477	2879	3319	3701	4127	4549
61	337	647	997	1361	1721	2111	2503	2887	3323	3709	4129	4561
67	347	653	1009	1367	1723	2113	2521	2897	3329	3719	4133	4567
71	349	659	1013	1373	1733	2129	2531	2903	3331	3727	4139	4583
73	353	661	1019	1381	1741	2131	2539	2909	3343	3733	4153	4591
79	359	673	1021	1399	1747	2137	2543	2917	3347	3739	4157	4597
83	367	677	1031	1409	1753	2141	2549	2927	3359	3761	4159	4603
89	373	683	1033	1423	1759	2143	2551	2939	3361	3767	4177	4621
97	379	691	1039	1427	1777	2153	2557	2953	3371	3769	4201	4637
101	383	701	1049	1429	1783	2161	2579	2957	3373	3779	4211	4639
103	389	709	1051	1433	1787	2179	2591	2963	3389	3793	4217	4643
107	397	719	1061	1439	1789	2203	2593	2969	3391	3797	4219	4649
109	401	727	1063	1447	1801	2207	2609	2971	3407	3803	4229	4651
113	409	733	1069	1451	1811	2213	2617	2999	3413	3821	4231	4657
127	419	739	1087	1453	1823	2221	2621	3001	3433	3823	4241	4663
131	421	743	1091	1459	1831	2237	2633	3011	3449	3833	4243	4673
137	431	751	1093	1471	1847	2239	2647	3019	3457	3847	4253	4679
139	433	757	1097	1481	1861	2243	2657	3023	3461	3851	4259	4691
149	439	761	1103	1483	1867	2251	2659	3037	3463	3853	4261	4703
151	443	769	1109	1487	1871	2267	2663	3041	3467	3863	4271	4721
157	449	773	1117	1489	1873	2269	2671	3049	3469	3877	4273	4723
163	457	787	1123	1493	1877	2273	2677	3061	3491	3881	4283	4729
167	461	797	1129	1499	1879	2281	2683	3067	3499	3889	4289	4733
173	463	809	1151	1511	1889	2287	2687	3079	3511	3907	4297	4751
179	467	811	1153	1523	1901	2293	2689	3083	3517	3911	4327	4759
181	479	821	1163	1531	1907	2297	2693	3089	3527	3917	4337	4783
191	487	823	1171	1543	1913	2309	2699	3109	3529	3919	4339	4787
193	491	827	1181	1549	1931	2311	2707	3119	3533	3923	4349	4789
197	499	829	1187	1553	1933	2333	2711	3121	3539	3929	4357	4793
199	503	839	1193	1559	1949	2339	2713	3137	3541	3931	4363	4799
211	509	853	1201	1567	1951	2341	2719	3163	3547	3943	4373	4801
223	521	857	1213	1571	1973	2347	2729	3167	3557	3947	4391	4813
227	523	859	1217	1579	1979	2351	2731	3169	3559	3967	4397	4817
229	541	863	1223	1583	1987	2357	2741	3181	3571	3989	4409	4831

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4861	5281	5701	6143	6577	7001	7507	7927	8389	8837	8293	9739
4871	5297	5711	6151	6581	7013	7517	7933	8419	8839	8297	9743
4877	5303	5717	6163	6599	7019	7523	7937	8423	8849	8311	9749
4889	5309	5737	6173	6607	7027	7529	7949	8429	8861	8317	9767
4903	5323	5741	6197	6619	7039	7537	7951	8431	8863	8329	9769
4909	5333	5743	6199	6637	7043	7541	7963	8443	8867	8353	9781
4919	5347	5749	6203	6653	7057	7547	7993	8447	8887	8363	9787
4931	5351	5779	6211	6659	7069	7549	8009	8461	8893	8369	9791
4933	5381	5783	6217	6661	7079	7559	8011	8467	8923	8377	9803
4937	5387	5791	6221	6673	7103	7561	8017	8501	8929	8387	9811
4943	5393	5801	6229	6679	7109	7573	8039	8513	8933	8389	9817
4951	5399	5807	6247	6689	7121	7577	8053	8521	8941	8419	9829
4957	5407	5813	6257	6691	7127	7583	8059	8527	8951	8423	9833
4967	5413	5821	6263	6701	7129	7589	8069	8537	8963	8429	9839
4969	5417	5827	6269	6703	7151	7591	8081	8539	8969	8431	9851
4973	5419	5839	6271	6709	7159	7603	8087	8543	8971	8443	9857
4987	5431	5843	6277	6719	7177	7607	8089	8563	8999	8447	9859
4993	5437	5849	6287	6733	7187	7621	8093	8573	9001	8461	9871
4999	5441	5851	6299	6737	7193	7639	8101	8581	9007	8467	9883
5003	5443	5857	6301	6761	7207	7643	8111	8597	9011	8501	9887
5009	5449	5861	6311	6763	7211	7649	8117	8599	9013	8513	9901
5011	5471	5867	6317	6779	7213	7669	8123	8609	9029	8521	9907
5021	5477	5869	6323	6781	7219	7673	8147	8623	9041	8527	9923
5023	5479	5879	6329	6791	7229	7681	8161	8627	9043	8537	9929
5039	5483	5881	6337	6793	7237	7687	8167	8629	9049	8539	9931
5051	5501	5897	6343	6803	7243	7691	8171	8641	9059	8543	9941
5059	5503	5903	6353	6823	7247	7699	8179	8647	9067	8563	9949
5077	5507	5923	6359	6827	7253	7703	8191	8663	9091	8573	9967
5081	5519	5927	6361	6829	7283	7717	8209	8669	9103	8581	9973
5087	5521	5939	6367	6833	7297	7723	8219	8677	9109	8597	10,007
5099	5527	5953	6373	6841	7307	7727	8221	8681	9127	8599	10,009
5101	5531	5981	6379	6857	7309	7741	8231	8689	9133	8609	10,037
5107	5557	5987	6389	6863	7321	7753	8233	8693	9137	8623	10,039
5113	5563	6007	6397	6869	7331	7757	8237	8699	9151	8627	10,061
5119	5569	6011	6421	6871	7333	7759	8243	8707	9157	8629	10,067
5147	5573	6029	6427	6883	7349	7789	8263	8713	9161	8641	10,069
5153	5581	6037	6449	6899	7351	7793	8269	8719	9173	8647	10,079
5167	5591	6043	6451	6907	7369	7817	8273	8731	9181	8663	10,091
5171	5623	6047	6469	6911	7393	7823	8287	8737	9187	8669	10,093
5179	5639	6053	6473	6917	7411	7829	8291	8741	9199	8677	10,099
5189	5641	6067	6481	6947	7417	7841	8293	8747	9203	8681	10,103
5197	5647	6073	6491	6949	7433	7853	8297	8753	9209	8689	10,111
5209	5651	6079	6521	6959	7451	7867	8311	8761	9221	8693	10,133
5227	5653	6089	6529	6961	7457	7873	8317	8779	9227	8699	10,139
5231	5657	6091	6547	6967	7459	7877	8329	8783	9239	8707	10,141
5233	5659	6101	6551	6971	7477	7879	8353	8803	9241	8713	10,151
5237	5669	6113	6553	6977	7481	7883	8363	8807	9257	8719	10,159
5261	5683	6121	6563	6983	7487	7901	8369	8819	9277	8731	10,163
5273	5689	6131	6569	6991	7489	7907	8377	8821	9281	8737	10,169
5279	5693	6133	6571	6997	7499	7919	8387	8831	9283	8741	10,177

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Numbers & their Prime Factors

N	Factors	*N*	Factors	*N*	Factors	*N*	Factors	*N*	Factors
1		51	3.17	101	p	151	p	201	3.67
2	p	52	$2^2.13$	102	2.3.17	152	$2^3.19$	202	2.101
3	p	53	p	103	p	153	$3^2.17$	203	7.29
4	2^2	54	2.3^3	104	$2^3.13$	154	2.7.11	204	$2^2.3.17$
5	p	55	5.11	105	3.5.7	155	5.31	205	5.41
6	2.3	56	$2^3.7$	106	2.53	156	$2^2.3.13$	206	2.103
7	p	57	3.19	107	p	157	p	207	$3^2.23$
8	2^3	58	2.29	108	$2^2.3^3$	158	2.79	208	$2^4.13$
9	3^2	59	p	109	p	159	3.53	209	11.19
10	2.5	60	$2^2.3.5$	110	2.5.11	160	$2^5.5$	210	2.3.5.7
11	p	61	p	111	3.37	161	7.23	211	p
12	$2^2.3$	62	2.31	112	$2^4.7$	162	2.3^4	212	$2^2.53$
13	p	63	$3^2.7$	113	p	163	p	213	3.71
14	2.7	64	2^6	114	2.3.19	164	$2^2.41$	214	2.107
15	3.5	65	5.13	115	5.23	165	3.5.11	215	5.43
16	2^4	66	2.3.11	116	$2^2.29$	166	2.83	216	$2^3.3^3$
17	p	67	p	117	$3^2.13$	167	p	217	7.31
18	2.3^2	68	$2^2.17$	118	2.59	168	$2^3.3.7$	218	2.109
19	p	69	3.23	119	7.17	169	13^2	219	3.73
20	$2^2.5$	70	2.5.7	120	$2^3.3.5$	170	2.5.17	220	$2^2.5.11$
21	3.7	71	p	121	11^2	171	$3^2.19$	221	13.17
22	2.11	72	$2^3.3^2$	122	2.61	172	$2^2.43$	222	2.3.37
23	p	73	p	123	3.41	173	p	223	p
24	$2^3.3$	74	2.37	124	$2^2.31$	174	2.3.29	224	$2^5.7$
25	5^2	75	3.5^2	125	5^3	175	$5^2.7$	225	$3^2.5^2$
26	2.13	76	$2^2.19$	126	$2.3^2.7$	176	$2^4.11$	226	2.113
27	3^3	77	7.11	127	p	177	3.59	227	p
28	$2^2.7$	78	2.3.13	128	2^7	178	2.89	228	$2^2.3.19$
29	p	79	p	129	3.43	179	p	229	p
30	2.3.5	80	$2^4.5$	130	2.5.13	180	$2^2.3^2.5$	230	2.5.23
31	p	81	3^4	131	p	181	p	231	3.7.11
32	2^5	82	2.41	132	$2^2.3.11$	182	2.7.13	232	$2^3.29$
33	3.11	83	p	133	7.19	183	3.61	233	p
34	2.17	84	$2^2.3.7$	134	2.67	184	$2^3.23$	234	$2.3^2.13$
35	5.7	85	5.17	135	$3^3.5$	185	5.37	235	5.47
36	$2^2.3^2$	86	2.43	136	$2^3.17$	186	2.3.31	236	$2^2.59$
37	p	87	3.29	137	p	187	11.17	237	3.79
38	2.19	88	$2^3.11$	138	2.3.23	188	$2^2.47$	238	2.7.17
39	3.13	89	p	139	p	189	$3^3.7$	239	p
40	$2^3.5$	90	$2.3^2.5$	140	$2^2.5.7$	190	2.5.19	240	$2^4.3.5$
41	p	91	7.13	141	3.47	191	p	241	p
42	2.3.7	92	$2^2.23$	142	2.71	192	$2^6.3$	242	2.11^2
43	p	93	3.31	143	11.13	193	p	243	3^5
44	$2^2.11$	94	2.47	144	$2^4.3^2$	194	2.97	244	$2^2.61$
45	$3^2.5$	95	5.19	145	5.29	195	3.5.13	245	5.7^2
46	2.23	96	$2^5.3$	146	2.73	196	$2^2.7^2$	246	2.3.41
47	p	97	p	147	3.7^2	197	p	247	13.19
48	$2^4.3$	98	2.7^2	148	$2^2.37$	198	$2.3^2.11$	248	$2^3.31$
49	7^2	99	$3^2.11$	149	p	199	p	249	3.83
50	2.5^2	100	$2^2.5^2$	150	$2.3.5^2$	200	$2^3.5^2$	250	2.5^3

The columns list for the numbers *N* (except 1) whether it is prime (**p**) or what its prime factors are. The dot (**.**) indicates multiplication.

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Value of 2^n	n =	Value of 2^n	n =
2	**1**	2 251 799 813 685 248	**51**
4	**2**	4 503 599 627 370 496	**52**
8	**3**	9 007 199 254 740 992	**53**
16	**4**	18 014 398 509 481 984	**54**
32	**5**	36 028 797 018 963 968	**55**
64	**6**	72 057 594 037 927 936	**56**
128	**7**	144 115 188 075 855 872	**57**
256	**8**	288 230 376 151 711 744	**58**
512	**9**	576 460 752 303 423 488	**59**
1 024	**10**	1 152 921 504 606 846 976	**60**
2 048	**11**	2 305 843 009 213 693 952	**61**
4 096	**12**	4 611 686 018 427 387 904	**62**
8 192	**13**	9 223 372 036 854 775 808	**63**
16 384	**14**	18 446 744 073 709 551 616	**64**
32 768	**15**	36 893 488 147 419 103 232	**65**
65 536	**16**	73 786 976 294 838 206 464	**66**
131 072	**17**	147 573 952 589 676 412 928	**67**
262 144	**18**	295 147 905 179 352 825 856	**68**
524 288	**19**	590 295 810 358 705 651 712	**69**
1 048 576	**20**	1 180 591 620 717 411 303 424	**70**
2 097 152	**21**	2 361 183 241 434 822 606 848	**71**
4 194 304	**22**	4 722 366 482 869 645 213 696	**72**
8 388 608	**23**	9 444 732 965 739 290 427 392	**73**
16 777 216	**24**	18 889 465 931 478 580 854 784	**74**
33 554 432	**25**	37 778 931 862 957 161 709 568	**75**
67 108 864	**26**	75 557 863 725 914 323 419 136	**76**
134 217 728	**27**	151 115 727 451 828 646 838 272	**77**
268 435 456	**28**	302 231 454 903 657 293 676 544	**78**
536 870 912	**29**	604 462 909 807 314 587 353 088	**79**
1 073 741 824	**30**	1 208 925 819 614 629 174 706 176	**80**
2 147 483 648	**31**	2 417 851 639 229 258 349 412 352	**81**
4 294 967 296	**32**	4 835 703 278 458 516 698 824 704	**82**
8 589 934 592	**33**	9 671 406 556 917 033 397 649 408	**83**
17 179 869 184	**34**	19 342 813 113 834 066 795 298 816	**84**
34 359 738 368	**35**	38 685 626 227 668 133 590 597 632	**85**
68 719 476 736	**36**	77 371 252 455 336 267 181 195 264	**86**
137 438 953 472	**37**	154 742 504 910 672 534 362 390 528	**87**
274 877 906 944	**38**	309 485 009 821 345 068 724 781 056	**88**
549 755 813 888	**39**	618 970 019 642 690 137 449 562 112	**89**
1 099 511 627 776	**40**	1 237 940 039 285 380 274 899 124 224	**90**
2 199 023 255 552	**41**	2 475 880 078 570 760 549 798 248 448	**91**
4 398 046 511 104	**42**	4 951 760 157 141 521 099 596 496 896	**92**
8 796 093 022 208	**43**	9 903 520 314 283 042 199 192 993 792	**93**
17 592 186 044 416	**44**	19 807 040 628 566 084 398 385 987 584	**94**
35 184 372 088 832	**45**	39 614 081 257 132 168 796 771 975 168	**95**
70 368 744 177 664	**46**	79 228 162 514 264 337 593 543 950 336	**96**
140 737 488 355 328	**47**	158 456 325 028 528 675 187 087 900 672	**97**
281 474 976 710 656	**48**	316 912 650 057 057 350 374 175 801 344	**98**
562 949 953 421 312	**49**	633 825 300 114 114 700 748 351 602 688	**99**
1 125 899 906 842 624	**50**	1 267 650 600 228 229 401 496 703 205 376	**100**

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N	N^2	N^3	N^4	N^5	N^6
1	1	1	1	1	1
2	4	8	16	32	64
3	9	27	81	243	729
4	16	64	256	1 024	4 096
5	25	125	625	3 125	15 625
6	36	216	1 296	7 776	46 656
7	49	343	2 401	16 807	117 649
8	64	512	4 096	32 768	262 144
9	81	729	6 561	59 049	531 441
10	100	1 000	10 000	100 000	1 000 000
11	121	1 331	14 641	161 051	1 771 561
12	144	1 728	20 736	248 832	2 985 984
13	169	2 197	28 561	371 293	4 826 809
14	196	2 744	38 416	537 824	7 529 536
15	225	3 375	50 625	759 375	11 390 625
16	256	4 096	65 536	1 048 576	16 777 216
17	289	4 913	83 521	1 419 857	24 137 569
18	324	5 832	104 976	1 889 568	34 012 224
19	361	6 859	130 321	2 476 099	47 045 881
20	400	8 000	160 000	3 200 000	64 000 000
21	441	9 261	194 481	4 084 101	85 766 121
22	484	10 648	234 256	5 153 632	113 379 904
23	529	12 167	279 841	6 436 343	148 035 889
24	576	13 824	331 776	7 962 624	191 102 976
25	625	15 625	390 625	9 765 625	244 140 625
26	676	17 576	456 976	11 881 376	308 915 776
27	729	19 683	531 441	14 348 907	387 420 489
28	784	21 952	614 656	17 210 368	481 890 304
29	841	24 389	707 281	20 511 149	594 823 321
30	900	27 000	810 000	24 300 000	729 000 000
31	961	29 791	923 521	28 629 151	887 503 681
32	1 024	32 768	1 048 576	33 554 432	1 073 741 824
33	1 089	35 937	1 185 921	39 135 393	1 291 467 969
34	1 156	39 304	1 336 336	45 435 424	1 544 804 416
35	1 225	42 875	1 500 625	52 521 875	1 838 265 625
36	1 296	46 656	1 679 616	60 466 176	2 176 782 336
37	1 369	50 653	1 874 161	69 343 957	2 565 726 409
38	1 444	54 872	2 085 136	79 235 168	3 010 936 384
39	1 521	59 319	2 313 441	90 224 199	3 518 743 761
40	1 600	64 000	2 560 000	102 400 000	4 096 000 000
41	1 681	68 921	2 825 761	115 856 201	4 750 104 241
42	1 764	74 088	3 111 696	130 691 232	5 489 031 744
43	1 849	79 507	3 418 801	147 008 443	6 321 363 049
44	1 936	85 184	3 748 096	164 916 224	7 256 313 856
45	2 025	91 125	4 100 625	184 528 125	8 303 765 625
46	2 116	97 336	4 477 456	205 962 976	9 474 296 896
47	2 209	103 823	4 879 681	229 345 007	10 779 215 329
48	2 304	110 592	5 308 416	254 803 968	12 230 590 464
49	2 401	117 649	5 764 801	282 475 249	13 841 287 201
50	2 500	125 000	6 250 000	312 500 000	15 625 000 000

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Random Numbers

33	16	17	80	55	00	24	70	65	47	13	98	45	48	07	58	65	16	27	86	40	96	99	96	71	87	02	92	24	67
59	29	83	78	16	98	44	63	63	44	62	29	61	77	36	41	57	30	33	72	69	44	03	71	40	49	45	23	85	47
28	42	94	06	55	88	78	46	57	89	03	07	67	69	57	26	28	39	83	43	95	47	35	02	43	60	54	54	77	08
68	22	74	72	86	75	81	49	24	46	08	81	75	93	02	10	03	68	33	33	17	18	62	44	47	84	81	06	50	51
00	45	73	51	24	81	48	77	85	02	78	31	29	92	82	33	09	75	52	26	94	76	71	69	87	95	28	12	60	11
64	65	57	68	32	77	80	89	01	07	98	82	61	42	32	33	77	51	82	86	16	20	22	80	09	17	54	23	09	27
82	19	15	27	41	98	87	81	76	10	78	15	23	99	16	73	69	28	62	78	72	65	17	85	51	69	02	06	23	50
04	24	91	21	28	30	13	58	21	37	23	96	83	15	02	77	56	75	34	98	89	28	02	36	80	42	12	63	13	58
08	19	55	43	08	80	25	47	77	70	96	36	65	50	76	39	69	47	93	52	69	77	41	12	85	30	44	14	19	07
55	85	22	22	74	18	46	19	38	60	55	87	26	12	13	04	42	00	35	24	70	56	97	87	80	84	44	71	25	34
94	02	50	10	16	02	94	62	48	30	80	11	89	92	32	96	01	71	42	30	13	48	19	70	49	40	20	65	19	90
32	14	70	92	20	85	83	91	37	06	26	76	93	33	72	46	38	02	40	03	88	62	55	33	32	65	28	99	82	59
15	00	21	86	78	87	19	04	77	12	16	73	02	68	99	48	76	43	74	62	83	03	94	17	75	10	63	45	87	46
08	07	84	34	92	74	12	22	28	17	08	88	74	86	10	11	02	60	85	35	65	18	23	03	02	19	94	04	33	21
38	24	93	32	27	15	88	09	44	18	00	29	90	30	33	77	81	55	54	20	49	34	33	63	02	26	25	71	35	35
56	47	95	65	87	58	79	63	02	04	66	08	58	71	76	81	05	58	28	14	66	41	73	63	09	39	43	04	84	34
30	37	63	25	08	82	04	15	60	13	06	92	01	94	14	75	67	54	33	63	74	20	63	66	98	54	07	78	78	59
22	11	03	22	87	88	82	07	66	17	04	77	83	94	92	20	01	65	06	81	29	14	50	95	86	86	37	31	59	85
10	95	52	34	85	22	41	67	47	54	05	97	33	96	96	45	61	27	50	53	96	95	26	43	91	63	37	45	22	14
87	99	81	64	07	27	89	61	57	76	38	63	44	76	60	75	36	55	20	07	86	71	80	09	37	62	07	45	40	75
86	38	15	36	24	88	99	11	47	70	58	47	56	59	16	77	37	54	37	68	12	95	19	52	89	44	93	15	22	74
46	99	41	04	84	36	41	89	59	20	73	72	66	92	54	52	23	34	56	57	30	85	59	21	83	29	27	33	38	03
78	63	59	18	21	65	97	12	74	71	02	75	08	89	04	64	34	77	17	67	61	06	58	41	64	19	00	93	35	99
64	79	78	54	00	86	46	33	97	97	47	95	79	99	65	35	03	65	48	55	11	90	83	24	05	76	70	90	13	98
26	65	14	46	63	60	23	15	31	31	85	04	71	65	93	15	77	51	98	29	80	29	28	55	36	43	21	94	09	88
23	56	33	62	19	27	58	02	29	37	27	92	15	64	90	02	86	45	55	50	89	76	02	81	99	66	31	76	31	37
83	05	65	98	89	13	82	63	73	24	31	66	26	47	90	00	15	49	61	22	43	84	11	21	07	84	38	53	21	65
65	18	35	94	69	49	95	32	57	77	42	39	83	33	83	90	40	56	80	19	61	77	63	47	04	15	26	40	77	30
29	12	38	59	98	66	49	20	95	69	45	82	96	68	95	70	08	83	00	32	06	25	07	14	28	70	72	71	92	72
49	59	57	61	61	92	63	55	27	64	00	28	89	00	50	33	91	92	24	28	92	63	16	18	12	86	79	86	67	02
28	12	55	13	47	33	86	40	94	73	46	84	63	32	91	83	44	79	14	29	64	58	37	69	33	01	10	94	10	17
82	59	47	84	59	55	13	51	27	46	83	62	28	11	78	12	04	35	83	07	65	05	41	95	59	85	90	17	42	61
83	97	59	46	90	18	43	96	96	48	05	02	82	36	75	79	85	02	31	88	07	69	77	32	07	62	70	22	18	88
10	40	11	27	11	85	71	19	59	59	00	27	54	50	04	25	31	99	08	23	03	53	39	58	73	62	54	92	22	12
10	17	48	32	15	50	90	18	21	37	21	77	76	38	96	24	72	44	37	85	60	89	54	75	93	17	95	08	24	07
39	99	29	56	78	72	19	99	98	58	74	77	69	62	37	86	43	35	17	24	76	99	92	31	06	64	01	78	46	83
32	38	12	75	87	18	22	70	45	76	69	28	31	70	18	67	96	03	38	16	54	98	11	33	07	83	61	54	94	43
58	29	28	80	04	72	96	25	13	71	26	23	14	59	36	58	79	84	85	21	93	89	23	79	43	20	39	56	82	68
96	66	64	58	05	05	17	43	90	00	84	05	74	53	67	75	01	35	96	56	36	62	40	68	37	98	17	02	08	84
61	27	19	48	76	93	16	94	85	80	01	95	36	66	31	19	69	08	68	53	89	95	73	50	14	94	29	79	33	92
38	98	42	54	96	22	27	77	54	00	29	26	40	92	12	06	20	03	13	70	40	24	83	44	17	16	14	80	02	34
46	02	52	02	56	82	58	11	49	12	71	47	72	93	52	55	36	37	53	57	52	94	69	71	99	15	82	63	86	79
18	16	56	86	81	38	05	33	20	57	45	50	99	45	57	24	48	86	19	73	74	04	08	03	58	27	60	41	29	44
59	73	79	36	07	34	59	27	24	30	12	62	57	94	20	29	86	31	40	17	93	75	84	67	84	38	49	13	09	44
31	61	29	14	08	08	01	01	97	94	43	05	68	60	18	87	09	20	81	47	81	70	92	37	26	68	22	83	35	63
57	47	61	57	46	26	38	74	80	98	28	50	95	70	90	72	20	31	56	41	05	85	85	79	48	90	22	39	45	00
89	88	28	62	21	44	33	20	80	46	78	48	66	57	83	79	30	31	26	68	04	35	31	76	72	55	53	77	96	84
84	02	82	12	34	42	33	54	03	49	88	67	86	83	41	09	40	13	92	60	47	11	67	93	31	43	76	32	60	91
69	54	54	10	97	76	38	99	31	11	08	05	64	39	67	26	87	28	08	26	17	81	98	03	75	79	08	20	09	31
81	84	64	61	85	67	08	84	21	56	37	46	31	22	30	96	24	99	27	55	89	39	18	26	10	55	53	84	73	37

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27	41	37	89	28	65	53	01	80	68	89	19	14	19	68	42	72	63	00	92	97	54	32	95	51	51	53	54	00	53
57	04	77	11	97	66	39	69	86	66	31	92	43	13	83	08	46	60	50	46	03	29	82	69	77	30	69	50	43	22
81	23	30	37	49	29	20	84	75	15	09	21	91	95	45	87	42	71	11	20	11	29	30	56	44	88	74	45	56	88
42	22	14	39	61	43	75	08	37	53	48	48	65	18	81	99	17	43	34	60	83	32	88	24	14	07	14	55	64	35
34	88	63	75	31	23	61	82	88	69	52	16	81	89	66	40	33	17	72	24	72	88	86	29	03	83	55	04	15	90
61	40	36	54	47	05	75	17	52	68	81	46	06	87	26	50	44	59	60	70	52	43	87	68	42	24	89	19	97	89
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59	22	47	04	30	30	36	00	25	82	25	64	12	72	08	25	71	16	86	24	15	42	69	70	64	39	34	32	42	79
49	37	02	86	66	16	18	27	40	13	25	57	21	88	69	79	29	55	98	13	61	92	57	45	53	34	76	08	89	27
92	43	33	24	77	67	83	80	00	56	91	49	34	60	11	85	08	49	72	49	08	09	15	95	76	20	45	73	57	95
80	77	16	39	20	35	09	87	41	31	22	29	19	31	74	43	18	08	62	84	79	09	26	84	52	24	95	22	76	29
31	04	68	94	04	62	22	85	08	69	52	99	24	89	39	81	67	28	43	50	71	46	52	78	72	82	13	32	46	61
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97	65	13	63	23	93	51	79	01	54	45	03	76	32	88	17	78	55	39	54	18	36	13	69	35	91	22	11	17	52
12	08	66	35	85	72	48	41	49	65	34	25	19	76	84	97	23	99	56	90	39	76	77	34	86	57	70	51	62	58
61	81	60	52	49	64	99	28	16	29	52	05	67	06	01	17	30	31	60	55	29	65	12	87	58	23	25	70	76	73
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79	47	72	80	07	79	86	08	14	65	25	14	85	13	64	13	16	95	71	29	77	06	23	98	63	29	19	70	00	68
68	75	89	83	80	16	03	99	25	59	29	96	37	69	38	76	85	47	61	28	46	23	88	63	20	60	06	61	25	43
50	62	65	80	55	33	96	09	93	13	66	72	65	70	22	82	69	52	64	54	24	45	80	25	33	65	80	56	21	29
76	56	30	52	10	30	80	48	34	52	99	34	20	71	56	31	25	05	73	75	51	87	16	68	16	59	65	44	25	97
62	23	06	29	72	49	26	76	15	21	35	96	34	02	21	86	22	01	67	50	11	78	71	57	70	22	68	35	98	23
67	08	40	88	36	77	91	77	35	01	74	95	12	55	30	32	91	32	08	91	60	51	96	52	45	88	00	57	56	08
53	34	50	19	99	45	88	42	18	57	40	78	79	92	08	52	29	45	62	30	05	46	48	29	40	60	87	12	56	57
05	78	53	99	85	44	20	18	87	55	75	13	93	80	42	45	15	28	25	74	57	13	57	77	42	98	86	24	38	51
32	73	55	70	30	22	16	16	87	38	63	20	85	97	17	78	17	66	85	96	60	41	99	13	95	06	76	90	56	48
00	39	70	29	21	35	18	55	44	66	72	18	39	78	71	92	25	75	04	88	31	78	76	70	99	43	22	67	71	36
53	63	28	77	02	37	34	76	27	10	02	07	37	80	39	43	75	59	48	95	48	23	86	08	65	91	01	26	90	82
50	50	30	93	22	08	32	39	17	15	72	30	23	71	84	78	74	47	00	91	09	78	59	03	62	97	61	39	26	93
03	73	46	75	98	79	36	55	02	49	82	95	05	16	29	72	57	03	96	03	97	41	31	28	62	62	02	88	06	06
88	78	02	73	62	12	81	24	62	49	34	87	61	28	37	03	43	62	58	08	00	52	61	32	46	88	98	21	28	84
01	33	29	93	00	04	59	04	10	71	42	94	40	98	99	37	92	12	86	32	29	96	56	28	32	76	77	69	71	04
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76	17	63	38	06	64	31	76	10	81	63	00	35	28	25	36	45	98	42	16	89	04	66	67	63	96	99	59	67	30
50	88	90	29	37	35	81	83	46	13	81	81	22	02	13	66	87	04	88	60	42	10	30	82	65	30	92	06	76	52
40	97	09	08	39	62	58	55	75	50	36	43	44	32	13	47	86	72	08	97	99	15	59	97	52	78	20	57	74	30
56	80	79	92	88	04	06	60	37	21	99	96	31	88	21	79	68	78	66	98	21	35	18	86	96	82	55	94	52	42
41	10	41	46	85	40	20	17	44	34	27	65	77	24	75	63	64	23	89	08	85	27	69	44	50	95	05	06	83	64
84	46	16	69	74	71	92	91	49	51	75	18	21	10	31	76	71	08	65	07	00	60	55	11	43	43	46	64	98	92
59	20	49	63	20	18	78	90	12	58	65	47	12	13	80	78	94	76	60	37	59	43	61	75	10	73	97	67	10	26
89	63	85	84	48	35	98	82	64	23	97	45	21	29	97	63	08	81	88	35	76	72	76	74	67	67	91	35	42	37
93	04	20	40	12	79	25	83	40	91	65	03	40	03	41	18	43	09	27	63	25	86	10	65	39	03	50	67	02	19
02	77	97	20	12	67	55	27	12	32	90	70	14	54	94	45	37	38	73	35	79	46	49	52	15	09	62	71	31	89

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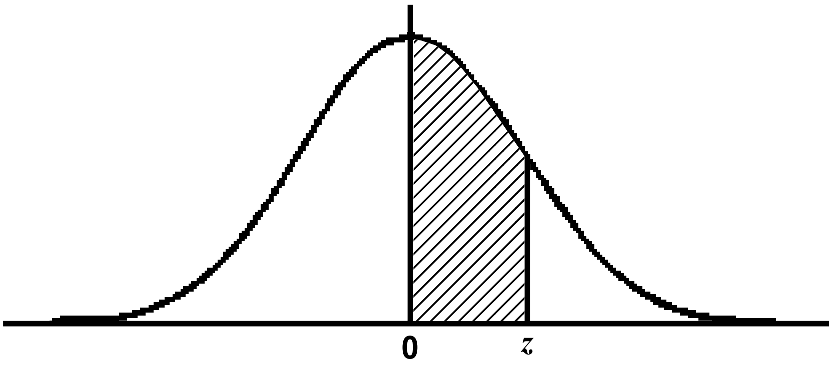
Areas under Curve of Normal Distribution



The table gives the fraction of the total area under the curve for the shaded area shown, which lies between the middle ordinate (*the mean*) and the ordinate at z for values of z from 0.00 to 3.99

(All values rounded to 4 decimal places)

↓z→	0	1	2	3	4	5	6	7	8	9
0.0	0.0000	0.0040	0.0080	0.0120	0.0160	0.0199	0.0239	0.0279	0.0319	0.0359
0.1	0.0398	0.0438	0.0478	0.0517	0.0557	0.0596	0.0636	0.0675	0.0714	0.0754
0.2	0.0793	0.0832	0.0871	0.0910	0.0948	0.0987	0.1026	0.1064	0.1103	0.1141
0.3	0.1179	0.1217	0.1255	0.1293	0.1331	0.1368	0.1406	0.1443	0.1480	0.1517
0.4	0.1554	0.1591	0.1628	0.1664	0.1700	0.1736	0.1772	0.1808	0.1844	0.1879
0.5	0.1915	0.1950	0.1985	0.2019	0.2054	0.2088	0.2123	0.2157	0.2190	0.2224
0.6	0.2258	0.2291	0.2324	0.2357	0.2389	0.2422	0.2454	0.2486	0.2518	0.2549
0.7	0.2580	0.2612	0.2642	0.2673	0.2704	0.2734	0.2764	0.2794	0.2823	0.2852
0.8	0.2881	0.2910	0.2939	0.2967	0.2996	0.3023	0.3051	0.3078	0.3106	0.3133
0.9	0.3159	0.3186	0.3212	0.3238	0.3264	0.3289	0.3315	0.3340	0.3365	0.3389
1.0	0.3413	0.3438	0.3461	0.3485	0.3508	0.3531	0.3554	0.3577	0.3599	0.3621
1.1	0.3643	0.3665	0.3686	0.3708	0.3729	0.3749	0.3770	0.3790	0.3810	0.3830
1.2	0.3849	0.3869	0.3888	0.3907	0.3925	0.3944	0.3962	0.3980	0.3997	0.4015
1.3	0.4032	0.4049	0.4066	0.4082	0.4099	0.4115	0.4131	0.4147	0.4162	0.4177
1.4	0.4192	0.4207	0.4222	0.4236	0.4251	0.4265	0.4279	0.4292	0.4306	0.4319
1.5	0.4332	0.4345	0.4357	0.4370	0.4382	0.4394	0.4406	0.4418	0.4429	0.4441
1.6	0.4452	0.4463	0.4474	0.4484	0.4495	0.4505	0.4515	0.4525	0.4535	0.4545
1.7	0.4554	0.4564	0.4573	0.4582	0.4591	0.4599	0.4608	0.4616	0.4625	0.4633
1.8	0.4641	0.4649	0.4656	0.4664	0.4671	0.4678	0.4686	0.4693	0.4699	0.4706
1.9	0.4713	0.4719	0.4726	0.4732	0.4738	0.4744	0.4750	0.4756	0.4761	0.4767
2.0	0.4772	0.4778	0.4783	0.4788	0.4793	0.4798	0.4803	0.4808	0.4812	0.4817
2.1	0.4821	0.4826	0.4830	0.4834	0.4838	0.4842	0.4846	0.4850	0.4854	0.4857
2.2	0.4861	0.4864	0.4868	0.4871	0.4875	0.4878	0.4881	0.4884	0.4887	0.4890
2.3	0.4893	0.4896	0.4898	0.4901	0.4904	0.4906	0.4909	0.4911	0.4913	0.4916
2.4	0.4918	0.4920	0.4922	0.4925	0.4927	0.4929	0.4931	0.4932	0.4934	0.4936
2.5	0.4938	0.4940	0.4941	0.4943	0.4945	0.4946	0.4948	0.4949	0.4951	0.4952
2.6	0.4953	0.4955	0.4956	0.4957	0.4959	0.4960	0.4961	0.4962	0.4963	0.4964
2.7	0.4965	0.4966	0.4967	0.4968	0.4969	0.4970	0.4971	0.4972	0.4973	0.4974
2.8	0.4974	0.4975	0.4976	0.4977	0.4977	0.4978	0.4979	0.4979	0.4980	0.4981
2.9	0.4981	0.4982	0.4982	0.4983	0.4984	0.4984	0.4985	0.4985	0.4986	0.4986
3.0	0.4987	0.4987	0.4987	0.4988	0.4988	0.4989	0.4989	0.4989	0.4990	0.4990
3.1	0.4990	0.4991	0.4991	0.4991	0.4992	0.4992	0.4992	0.4992	0.4993	0.4993
3.2	0.4993	0.4993	0.4994	0.4994	0.4994	0.4994	0.4994	0.4995	0.4995	0.4995
3.3	0.4995	0.4995	0.4995	0.4996	0.4996	0.4996	0.4996	0.4996	0.4996	0.4997
3.4	0.4997	0.4997	0.4997	0.4997	0.4997	0.4997	0.4997	0.4997	0.4997	0.4998
3.5	0.4998	0.4998	0.4998	0.4998	0.4998	0.4998	0.4998	0.4998	0.4998	0.4998
3.6	0.4998	0.4998	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999
3.7	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999
3.8	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999
3.9	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000

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Symbols and Abbreviations

Mathematics uses many symbols and abbreviations to represent instructions, or numbers, in a more concise form. Here, with a brief note as to their meaning, are the ones most commonly used.
see also The Greek Alphabet

$+$	add *or* plus *or* positive
$-$	minus *or* subtract *or* negative
$\sim$	find the absolute difference of
$\times$	times *or* multiplied by
*	times *or* multiplied by
$\div$	divided by
/	divided by
$\pm$	add or subtract plus or minus positive or negative
$=$	equals *or* is equal to
$\neq$	does not equal *or* is not equal to
$\approx$	is approximately equal to
$\equiv$	is equivalent to *or* has the same value as is identically equal to is congruent to
$<$	is less than
$\leqslant$	is less than or equal to
$>$	is greater than
$\geqslant$	is greater than or equal to
$\propto$	varies as *or* is proportional to
:	proportion
.	decimal (*or* fraction) point
,	decimal marker
%	per cent *or* out of a hundred
‰	per mil *or* out of a thousand
( )	brackets *or* parentheses
$\langle\ \rangle$	angle brackets
[ ]	square brackets
{ }	curly brackets *or* braces also used to enclose a set
$[x]$	the largest whole number which is not greater than x
$\lvert x \rvert$	the absolute value of x
x^2	x squared
x^3	x cubed
x^n	x to the nth power
$\sqrt{x}$	the square root of x
$\sqrt[3]{x}$	the cube root of x
$\angle$	angle
$\parallel$	is parallel to
$\nparallel$	is not parallel to
$\perp$	is perpendicular to
$^\circ$	degrees
$'$	minutes
$''$	seconds

$\mathbb{N}$	the set of **natural** numbers
$\mathbb{Z}$	the set of **whole** numbers
$\mathbb{Q}$	the set of **rational** numbers
$\mathbb{R}$	the set of **real** numbers
$\mathbb{C}$	the set of **complex** numbers
$\in$	is a member of
$\notin$	is not a member of
$\subset$	is a subset of
$\not\subset$	is not a subset of
$\supset$	includes
$\cup$	union
$\cap$	intersection
$\varnothing$	null *or* empty set
$\Rightarrow$	implies
$\Leftarrow$	is implied by
$\Leftrightarrow$	implies and is implied by
$\therefore$	therefore
∞	infinity
$n!$	factorial n
$!n$	sub-factorial *or* derangements of n
i	square root of $^{-}1$
e	$\approx 2.71828 \ldots$
π	$\approx 3.14159 \ldots$
$f(x)$	function of x
$f'(x)$	first derivative of $f(x)$
$\int$	integral *or* anti-derivative
&	hexadecimal number follows

AP	arithmetic progression
APR	annual percentage rate
cu	cubic *(referring to units of volume)*
dp	decimal places
g c d	greatest common denominator
h c f	highest common factor
l c d	lowest common denominator
l c m	lowest common multiple
m	gradient of a line
mod	modulus
QED	which was to be proved
s f	significant figures
sq	square *(referring to units of area)*
UT	Universal Time *(Greenwich Mean Time)*

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The Greek alphabet is a rich source of symbols used in both mathematics and science, to the extent that nearly every one of them (both capitals and lower case) is used in some way or other. Some of them appear more than once to represent different things. Below is the full alphabet, and the names of the various symbols. The capital form of the letter is given in the first column, followed by the lower case version and its name. Then some of the more commonly seen meanings of usage are given.

Α	α	alpha	α β γ are often used to identify angles in plane figures.
Β	β	beta	
Γ	γ	gamma	
Δ	δ	delta	Δ is sometimes used to represent the area of a plane figure. δ is used (in calculus) to show that a small amount is considered.
Ε	ε	epsilon	
Ζ	ζ	zeta	
Η	η	eta	
Θ	θ	theta	θ is used to indicate a general angle
Ι	ι	iota	
Κ	κ	kappa	
Λ	λ	lambda	λ is used to represent a scalar in vector work
Μ	μ	mu	μ is used (in the SI system) to represent the prefix *micro* μ is sometimes used to represent the arithmetic mean
Ν	ν	nu	
Ξ	ξ	xi	ξ is sometimes used as the symbol for the universal set
Ο	ο	omicron	
Π	π	pi	Π is used to show that a continued product is needed π is used to represent the value of the irrational number 3.14159 … $\pi(n)$ means the number of primes less than, or equal to n
Ρ	ρ	rho	
Σ	σ	sigma	Σ is used to show that the sum of a series is to be found σ is used to represent the standard deviation of a population
Τ	τ	tau	τ is used to represent the golden ratio 1.6180 … (see also phi)
Υ	υ	upsilon	
Φ	φ	phi	Φ is sometimes used as the symbol for the empty set ϕ is used to represent the golden ratio 1.6180 … (see also tau) $\phi(n)$ means the number of positive integers less than, and relatively prime to, n
Χ	χ	chi	χ is used in statistics in reference to the chi-squared test
Ψ	ψ	psi	
Ω	ω	omega	

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